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SIMULATION AND CONTROL OF A PRESSURISED HEAVY WATER REACTOR

A Thesis Submitted
In Partial Fulfilment of the Requirements
for the Degree of
DOCTOR OF PHILOSOPHY

by

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DEPARTMENT OF MECHANICAL ENGINEERING
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CERTIFICATE

Certified that this work entitled 'Simulation and Control of a Pressurised Heavy Water Reactor' has been done by Shri M.G. Subramanian under our supervision and has not been submitted elsewhere for a degree.

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NOMENCLATURE

a	–	area of valve opening
c	–	concentration of precursor
c'	–	normalised value of c
d	–	diameter
f	–	friction factor
h	–	specific enthalpy
k	–	thermal conductivity
ℓ	–	length
ℓ^*	–	neutron life time
n	–	neutron population
n'	–	normalised value of n
p	–	pressure (steam drum)
s	–	perimeter
t	–	time
v	–	specific volume
w	–	mass flow rate (per heat exchanger)
w'	–	normalised value of w
x	–	quality of two-phase mixture
y	–	level
F	–	area
H	–	heat transfer coefficient
M	–	mass

Q	-	heat flux
P	-	primary coolant circuit pressure
T	-	temperature
V	-	volume
$\underline{x}$	-	state vector
$\underline{u}$	-	control vector
$\underline{w}$	-	disturbance (uncontrollable) vector
α	-	temperature coefficient of reactivity
β	-	total delayed neutron fraction
β_i	-	delayed neutron fraction of group i
λ	-	decay constant for one-group model
λ_i	-	decay constant of i^{th} neutron precursor
δk	-	reactivity
ρ	-	density

Subscripts

o	-	initial condition
b	-	boiling leg
d	-	downcomer
f	-	saturated liquid
g	-	saturated vapour
i	-	i^{th} element, section or identification number
m	-	make up (feed) water
p	-	preheat leg

r	–	riser leg
s	–	steam
sat	–	at saturation
ro	–	reactor outlet
ri	–	reactor inlet
af	–	average for fuel element
ac	–	average for coolant
fb	–	feedback
w1,w2,w3	–	properties of wall at riser, boiling and preheat leg
2,3,4	–	riser, boiling and preheat leg
pav	–	average for the primary heat transport circuit
D	–	primary heat transport circuit
mi	–	feedwater at inlet to preheat leg
mo	–	feedwater at outlet of boiling leg

Superscripts

o	–	initial condition
T	–	transpose of a vector or a matrix
$(\dot{\ })$	–	denotes differentiation with respect to time
$(\underline{\ })$	–	denotes a vector or a matrix

SYNOPSIS

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A major portion of the present-day energy requirements is met by hydroelectric and fossil fired thermal power stations. However, the increasing demand for power and the current oil crisis make it imperative to explore other sources of energy. Nuclear fission energy seems to be a promising alternative. Efficient operation of existing nuclear power plants and development of plants for future needs requires an understanding of the nuclear reactor and other auxiliary systems. In this work, modelling of a pressurised heavy water reactor is carried out. The model is then simulated on a digital computer to study its dynamic - uncontrolled and controlled - behaviour.

A reactor of the CANDU type - CANDU is a heavy water moderated, pressurised heavy water cooled natural uranium fuelled pressure tube reactor - is studied in this thesis. The system for producing steam comprises U-type shell and tube heat

exchangers, where primary coolant transfers heat to water and a steam drum. One leg of the shell of the heat exchanger forms the riser for natural circulation unit while the other forms part of the feed water circuit. A mathematical model represented by a set of coupled nonlinear, ordinary, differential equations is formulated for the above mentioned system - reactor, heat exchanger and steam drum with natural circulation loop - from physical principles. The system is then simulated on a digital computer to study its dynamic behaviour when operating at full load, it is acted upon by disturbances like step change in the area of the steam valve, step change in the temperature of the feed water, etc. In order to design controllers so that the average temperature of the coolant in the reactor, the pressure of the primary coolant circuit, and the water level in the steam drum are maintained at prescribed values, the nonlinear model is linearised by a first order Taylor series expansion around full load conditions. The resulting linear, coupled, ordinary differential equations with constant coefficients are then expressed in state space form to facilitate the application of optimal control theory. Uncontrolled responses of the nonlinear model are compared with those of the linear model for equivalent disturbances to validate the linearization procedure.

The problem of selection of controllers is then formulated as a linear regulator problem, where an objective function involving a quadratic function of states and control inputs is minimised subject to system's governing equations. Firstly a state feedback controller of proportional type - control inputs are linear, constant combinations of all states of the system - which drives the system states to prescribed values depending on magnitude of disturbance, is obtained. However, this is not suitable if the disturbances are not measurable or if the parameters of the system vary from the nominal values. To overcome these drawbacks, a proportional and integral type of controller which forces selected states to zero in the presence of constant disturbances is obtained. Such a controller requires a knowledge of all state variables. Since some of the state variables are not available for control purposes, a simpler control scheme based on available states is desirable. To achieve such a controller, the linear regulator problem is treated as a function minimization problem in the parameter space of the feedbacks, for which the necessary condition of optimality is that the gradient of the objective function with respect to each allowed feedback must vanish. Using explicit expressions for the value of the objective function and its gradient with respect to each allowable feedback, a descent method based on the gradients

is used to minimize the objective function and to achieve optimality. This approach leads to the design of controller of specified or preselected configuration and it is utilised to design analog controllers which minimize a performance index. Such a controller is implemented on the linear model. Controlled responses of the system with the simpler, easy-to-implement control scheme compare favourably with those where control scheme requires feedback of all the states. Lastly, the optimal analog controller obtained from the analysis of linear system is implemented on the nonlinear model and the controlled responses of the nonlinear model are provided for comparison.

CHAPTER 1

INTRODUCTION

A major portion of the present day energy requirements is met by hydroelectric and fossil fuelled thermal power stations. However increasing demand for power and the current oil crisis make it imperative to explore other forms of energy such as nuclear energy and solar energy. Nuclear fission energy seems to be a promising source for future electrical power generation. In the U.S.A., U.S.S.R., Canada, U.K. and other industrialised nations, a number of nuclear power plants are operating successfully. In India, two nuclear power stations are in operation and two more are being commissioned. The number of such installations is bound to increase rapidly in the foreseeable future with many countries opting for nuclear power.

1.1 ROLE OF COMPUTERS IN NUCLEAR POWER PLANTS

The expected increase in the number of nuclear power plants with a consequent increase in their sizes will place great emphasis on the development of instrumentation system that will increase plant availability and prevent outages. Functionally, the instrumentation provides data which can be used for fuel management, detection and correction of power distribution changes in the core, control of plant parameters like steam pressure, temperature, water-level in the steam drum etc. In most plants the above functions are accomplished by analog devices. However it has been felt that a computer

could be readily applied to the nuclear plant to perform many of the functions itemized above[1].

Today process-control computers are being increasingly used in the operation of nuclear power plants. Both start-up and shutdown routines and steady state operation are being entrusted to computers in some power stations[2]. In addition, a plant computer can relieve the operator of the tedious job of logging data and can simultaneously analyse the data if an operational problem exists. Of the 56 US thermal stations under construction in 1970, 48 had computers that performed digital monitoring, 33 had computers that calculated performance data and 14 had computers for starting the turbine[3]. The success achieved in conventional power stations has led to the use of computers in nuclear power plants, the trend being to utilise the computer to assimilate data, make calculations and as an operational aid. Its use in control and safety functions is generally avoided[4]. However power plants like Douglas Point Generating Station have used computers for direct digital control. The promising results obtained from Douglas Point G.S. have led to the application of computer systems in Ontario Hydro's Pickering G.S. and Bruce G.S. and Gentilly Nuclear G.S. in Canada[5]. When operating experience justifies their reliability, economy, and accuracy, nuclear plant computers will be used for various automatic control functions. In anticipation of this, efforts are being made to apply advanced

technology to the plant control system and studies[6] have been conducted for optimal digital control of nuclear reactors.

1.2 LITERATURE REVIEW

Analysis and design of optimal feedback control systems require a detailed understanding of the dynamical behaviour of the power station. The work of Chien et al.[7] is directed towards this objective. Chien et al. developed a mathematical model for a drum type natural circulation boiler. Dynamic behaviour of the boiler was described about a steady state operating level by perturbing the nonlinear mass, momentum and heat balance equations and the heat transfer equations. Uncontrolled transient responses were obtained for various inputs. Conventional analog controllers for closed loop control were selected based on a comparison of controlled responses of the plant with uncontrolled responses for equivalent disturbances. The work of Chien et al. initiated the use of digital computer for studying the dynamical behaviour of power plants.

Nicholson[8], following an analysis similar to that of Chien et al. obtained open loop responses of an oil-fired boiler. The order of the mathematical model was reduced analytically by neglecting the isolated higher order modes. Uncontrolled responses of the original model and the reduced model were compared. Using the reduced model, optimal digital control was obtained based on the minimisation of a performance index related to system's state variables and system inputs.

The dynamic behaviour of the system with optimal control was compared with the behaviour of the system with conventional analog control. In a subsequent work[9], turbo-alternator was included to study the combined boiler-turbo-alternator system. Performance of the system with optimal digital control and with conventional control was then investigated.

Anderson[10] studied the dynamic behaviour of a natural circulation boiler. Transient responses using conventional analog control, direct digital control and optimal digital control were obtained with a view to study the possibility of improving the performance of the plant by using an optimal control system. A similar study was performed by Anderson et al.[11] for a 200 MW boiler. In this analysis, random input disturbances and measurement noise were taken into account. Thompson[12] and Adams et al.[13] developed mathematical models for drum type boiler and once-through boiler respectively and the responses of the model agreed with the responses obtained from field tests. An IBM report[14] details the simulation models for hydro, thermal and nuclear power systems.

Dynamic analysis of a nuclear boiler based on a steam generating, pressure tube, heavy water moderated design was performed by M'Pherson et al.[15] A detailed mathematical model to represent plant dynamics was described with special reference to the boiling channel and steam drum dynamics. A simplified

model was derived for analogue computer studies. Effect of analog control systems on the performance of the plant was then studied.

Optimal control theory has been used to study the problems associated with the reactor such as optimal xenon shut down[16], xenon spatial oscillations[17], reactor start up and transient response. Lipinski[18] provides a detailed survey of the works done in optimal control of the reactor.

Kalley[19] suggested four applications of dynamic programming technique to nuclear reactor. These are: optimisation of poison distribution, optimisation of overall plant efficiency, design of optimal control programs and determination of flow distribution through a heat exchanger. Kalley outlined the optimal solution to a minimum energy start-up problem.

Ash[20] used dynamic programming to derive a functional equation which would cause a boiling water reactor to be driven to its equilibrium condition in minimum time by continuously moving the control rods.

Using Pontryagin's principle, Mohler[21] obtained the optimal open loop control to study the minimum time control of neutron density. Sekhar and Weaver[22] investigated optimal closed loop control of a reactor by using a set of equations linearised around the nominal trajectory alongwith the minimisation of a quadratic performance index.

Duncombe[23] defined a control variable which was equal to the product of flux density and reactivity and hence made the neutron kinetics equations linear. The performance index included a term of reactivity times flux squared. Optimal closed loop control was obtained by using the maximum principle for nuclear reactor load control. Lipinski[18] studied the optimal digital control of nuclear reactors with estimation of state variables. In the above studies, only the reactor kinetics with suitable approximations such as prompt jump, and one-group delayed neutron model was considered.

Ciechanowicz[24] investigated two types of digital control schemes for the linear model of the Halden Boiling Water Reactor; one involves the control strategy calculation for the overall dynamic system and in the other, the overall system was split into two systems characterised by smaller number of variables, the interaction between the two sub-systems being included by the use of cross-coupling controllers. The splitting produces the advantage of reduced computational effort. Dynamic behaviour of HBWR under different modes of control was compared with the behaviour of the reactor without control.

Bjirlo et al.[6] studied the transient response of the pressure in a boiling heavy water reactor. Experimental results which compare digital control and existing analog control for equivalent perturbations were provided. It was then concluded that digital control handles the perturbations effectively.

Ciechanowicz's work[25] is a combination of the earlier studies[6,24] for the investigation of multi-level control of a power plant. A 500 MW BWR power plant, treated as three coupled cores was studied for controlled and uncontrolled behaviour. Overall control strategy was investigated for different sampling periods of digital control and for the case where constraints were imposed on reactivity controller signals. Further, the influence of noise upon the overall dynamics of the power reactor was studied.

Slivinsky and Weaver[26] used the state variable technique to design non-interacting controllers for a system with m-inputs and m-outputs and illustrated the design of controllers for coupled core reactors where each core is controlled by the controller corresponding to that particular core.

Beneznai and Sinha[27,28] investigated the design of digital controllers with provision to adopt to changes in plant parameters. The response of the reactor was approximated by that of a low order model in a desired manner, the model parameters being chosen to minimize the deviation between the system and the model responses. Optimal feedback parameters for the model were then computed by means of the matrix Riccati equation.

1.3 OUTLINE AND SCOPE OF THE THESIS

In India, there are two nuclear power stations; the one at Tarapur has a boiling water reactor and the other in Rajasthan has a pressurised water reactor. A few plants similar to the one in Rajasthan are expected to be commissioned in the near future. The research described in this thesis was undertaken with the objective of applying modern control theory to design a control system for a pressurised water reactor. Chapter 1 is devoted to a review of the application of computers in power plants and of the optimal control of nuclear reactors.

A brief description of a pressurised heavy water reactor is made in Chapter 2. In Chapter 3 the nonlinear ordinary differential equations which describe the dynamic behaviour of the plant are derived from physical principles. Uncontrolled transient responses for selected disturbances are also obtained by simulating the nonlinear model on an IBM 7044 digital computer.

In Chapter 4, the nonlinear equations developed in the previous chapter are modified and are linearised around the steady state operating point at full power. This leads to the representation of the system by a set of linear ordinary differential equations. These equations are manipulated into state space form to yield a linear time invariant system for the reactor. Transient responses are then obtained for

disturbances identical to those assumed for the nonlinear model. A comparison of the uncontrolled responses of the two models is made to validate the linearisation procedure.

In Chapter 5, the control problem is formulated as a linear regulator problem with the objective function as a quadratic function in the state and control variables. Controllers of the proportional type and of the proportional and integral type are obtained. Behaviour of the plant with these controllers providing the feedback control are discussed when the plant is acted upon by constant disturbances.

The controller obtained in Chapter 5 assumes that all state variables are measurable and hence are available for the construction of the control law. This may not be possible in many physical situations. In such cases, a controller of preselected configuration is desirable. Therefore in Chapter 6, a controller whose structure is specified is obtained. The design of this controller is formulated as a function minimisation problem in the parameter space of the feedbacks. A descent technique based on the gradient of the performance index with respect to each allowable feedback is used to design the controller. Transient behaviour of the plant with this control is compared with that obtained in Chapter 5 for equivalent disturbances. Further, this controller is suitably modified and is

incorporated in the nonlinear model developed in Chapter 3 and the transient responses of the plant are discussed for constant disturbances.

Finally in Chapter 7 the results of the present work are summarised and suggestions for further work are provided.

CHAPTER 2

DESCRIPTION OF PRESSURISED HEAVY WATER REACTOR

2.1 INTRODUCTION

A conventional thermal power station and a nuclear power station differ mainly in the heat generating unit - the former utilising a furnace where chemical changes of the fuel (coal or oil) produce heat energy and the latter using a nuclear reactor where nuclear changes in the fuel liberate the heat. The heat thus liberated is used to produce steam in a steam generator. Most of the other equipments such as steam turbines and electric generators are quite alike for both the power plants. In this chapter brief descriptions of nuclear power reactors and more specifically a pressurised heavy water reactor of the CANDU type are presented.

2.2 DESCRIPTION OF NUCLEAR REACTOR

The basic features of a nuclear reactor are the reactor core in which heat is produced, a coolant system to remove the heat from the core and a heavy shield surrounding the core to contain all emanations that might otherwise be harmful to human life or health[29]. The core consists of a fuel which contains uranium, plutonium or thorium atoms. When a neutron is absorbed by a fuel nucleus, there is a finite probability of splitting this nucleus into fragments resulting in the release of two or more neutrons. This process is known as fission. The neutrons produced due to fission may take

part, in addition to causing further fission, in several reactions; they may be absorbed in core material other than the fuel or may be lost by leaking out of the core. For the fission process to be sustained, for each fuel nucleus capturing a neutron and undergoing fission, a minimum of one neutron, on the average, must be produced and absorbed by another fuel nucleus to cause fission. Since a large amount of energy (about 200 million electron volts or 0.32×10^{-10} Watt sec. per fission) is released in fission, a sustained fission process provides a steady heat source.

A salient feature of the fission process is that all neutrons are not released simultaneously. Most of the neutrons are emitted within about 10^{-12} seconds after fission. These are known as prompt neutrons. However, a small portion, less than one percent, of the total neutrons is given off an appreciable time after the fission has occurred and these neutrons are called delayed neutrons. These are emitted due to the radioactive decay of certain fission fragments, known as precursors. The half life of these precursors vary from 0.05 second to 55.6 seconds. The delayed neutrons are of tremendous importance since these facilitate the control of chain reaction, which otherwise would have been quite difficult, if not impossible.

Depending on the average speed of the neutrons in the reactor, reactors are classified as fast, intermediate and thermal reactors. Neutrons released in fission are at

high energies (i.e. have high velocities). Neutron energy on the average is greater than one million electron volt. When such neutrons cause most of the fissions, then such reactors are called as fast reactors. However, neutrons having energies less than about 0.1 electron volt have greater probability of causing fission than fast neutrons. In some reactors the fast neutrons are slowed down to thermal energies (0.025 electron volt) by collisions with certain materials, known as moderator, before they are absorbed by the fuel. Such reactors, where thermal neutrons maintain fission, are called thermal reactors and the fuel-moderator arrangement is known as core.

Heat generated in the core is removed by circulation of a coolant. Depending on the type of fuel, moderator and coolant,reactors are classified into various categories such as the boiling water reactor, pressurised water reactor, gas cooled reactor, etc.

A large number of commercial nuclear electrical power generating systems utilise water cooled and moderated thermal reactors[30]. Two basic types are generally used - pressurised water reactors which have no bulk boiling of coolant in the core and boiling type which permits bulk boiling in the core. Of the two operating atomic power stations in India, the one at Tarapur uses a boiling water reactor whereas the

reactor of Rajasthan Atomic Power Station (RAPS) is of the pressurised variety. Systems, similar to RAPS, are under construction at Madras and at Narora.

2.3 DESCRIPTION OF AN ATOMIC POWER STATION USING A PRESSURISED WATER REACTOR

An outline of the power station is as follows[31]:

The power station consists of a CANDU (Canadian Deutrium Uranium) type reactor supplying steam to a turbo-generator of 200 MW generating capacity. CANDU is a heavy water moderated, pressurised heavy water cooled, natural uranium fuelled pressure tube reactor. The reactor has a horizontal cylindrical tank and tube assembly. This assembly is known as calendria. Natural uranium in the form of pressed and sintered pellets is sheathed in Zircaloy tubes to form cylindrical fuel elements. Nineteen such elements are assembled to form a fuel bundle. The calendria tube contains thick walled coolant tube, having a fuel bundle and primary coolant, viz., heavy water at high pressure. The shell of the calendria contains heavy water at low pressure. This heavy water is used as moderator.

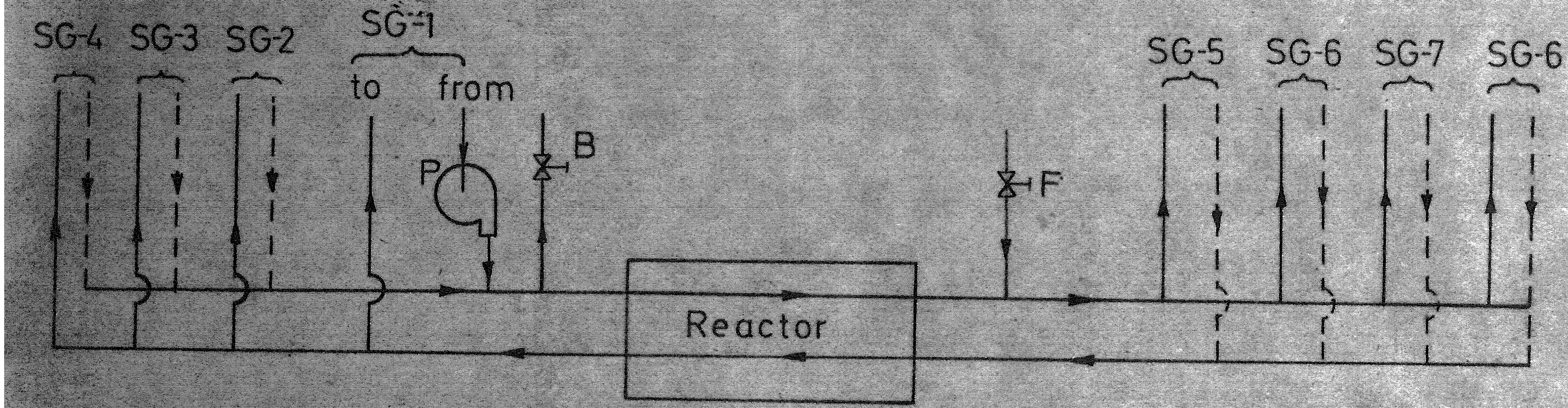
The two functions of the heavy water namely as a moderator and as a coolant are separate, each having its own closed circulating system. The fuel coolant system, called the primary heat transport system, is a high pressure and high temperature circuit with no bulk boiling. The

moderator circuit is a low temperature, low pressure circuit. These two are insulated from each other in the reactor core by a gas annulus between the calendria tube and the coolant tube.

Primary heat transport system transfers the heat generated in the fuel elements to heat exchangers where the heat is utilised to generate steam in a secondary coolant system. The primary system, shown in Fig.2.1, consists essentially of two identical loops in series, one at each end of the reactor. Each loop includes four boiler assemblies in parallel and four pumps in parallel. Such an arrangement circulates the primary coolant, through alternate coolant tube assemblies, in cpposite directions. Each boiler assembly consists of ten vertical, U type, tubes and shell heat exchangers in parallel and a steam drum.

The hot heavy water from the reactor enters the tube sides of the heat exchangers at the bottom of their recirculating sections after a transport delay of 5.6 seconds. Then it returns from the bottom of the preheater legs of the heat exchanger to the reactor. The transport delay associated with the flow from the steam generator to the reactor is of the order of 7.2 seconds.

The secondary coolant system, shown in Fig.2.2 consists of boiler feed water entering the shell side of the each heat



B - Bleed valve for heavy water

F - Feed valve for heavy water

SG - Steam generator (10 heat exchangers in parallel with a common steam drum)

P - Primary coolant circulation pump

FIG. 2.1 SCHEMATIC DIAGRAM OF PRIMARY COOLANT CIRCUIT

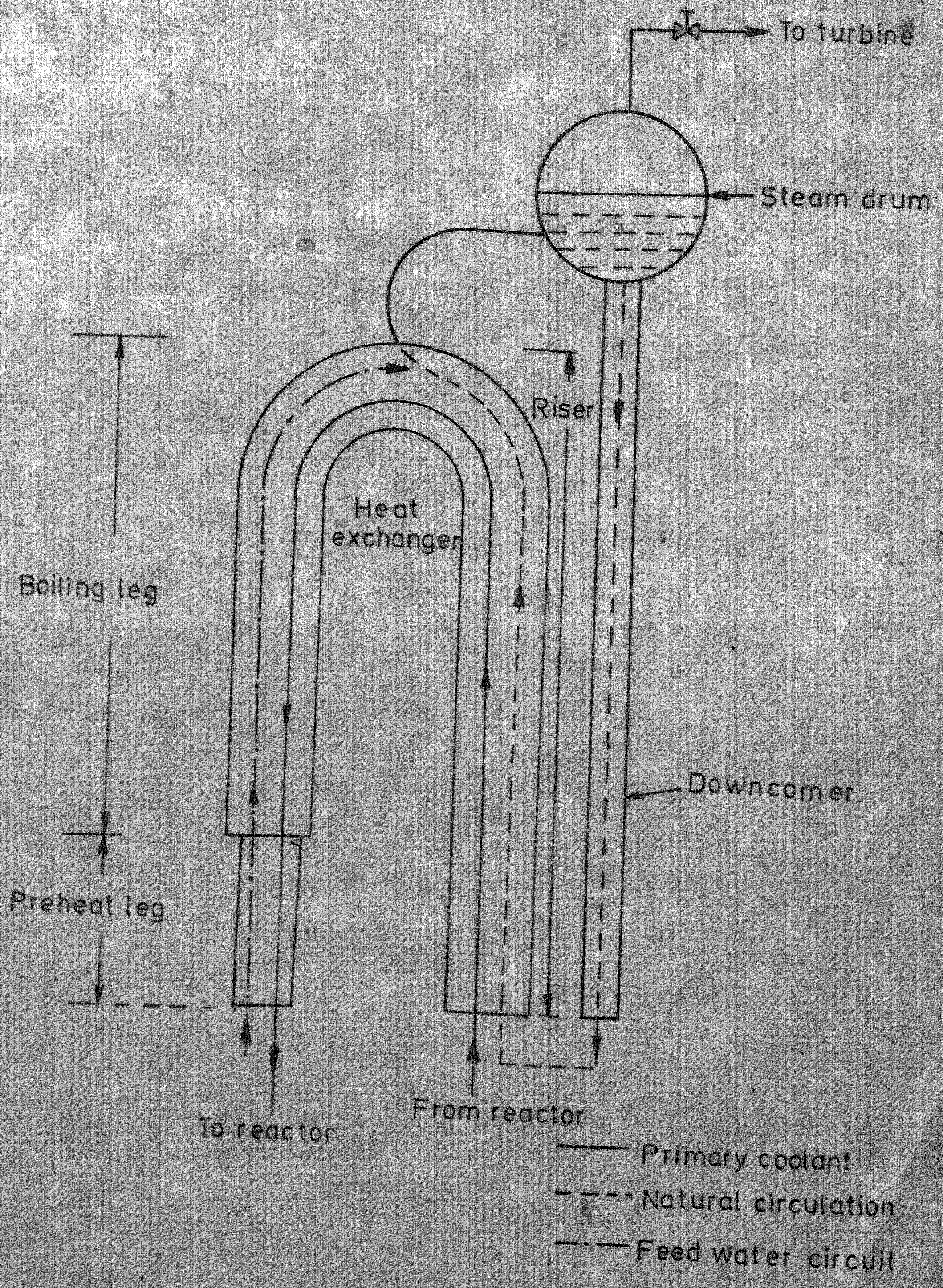


FIG. 2.2 SCHEMATIC DIAGRAM OF HEAT EXCHANGER AND STEAM DRUM

exchanger at the bottom of the preheater leg. The feed water is heated to or near the saturation temperature in the reduced diameter length of the leg. It boils in the upper portion of the leg and enters the steam drum above it. Here water and steam are separated. The water at saturation temperature flows through down-comers to the bottom of the heat exchanger's recirculating leg where it boils and returns to the steam drum. At full load, design pressure at the steam drum is 569 psig.

Saturated steam (0.26 percent maximum wet) from the steam drum is supplied to the turbine unit at 565 psig. The turbine has one single flow high pressure element and two double flow low pressure elements. The exhaust from the low pressure unit is condensed in a surface condenser. The condensate after six stages of heating, returns to the preheat leg of the heat exchanger, thereby completing the secondary coolant circuit.

The quantity of steam produced in the secondary system varies with reactor power. Power is controlled by adjusting the reactivity. This can be accomplished by changing the moderator level or by adjusting the neutron absorbing rods, called control rods, or by adding or removing a poison to the moderator. In the present study, however, it is assumed that reactivity changes are brought about by

movement of control rod only.

Pressure of the heavy-water coolant at reactor outlet is maintained at 1250 psia by feeding or bleeding heavywater to or from the primary heat transport system. Water level in the steam drum is controlled by adjusting the feedwater flow. The level of water is designed to vary linearly with load. But in this work, the water level in the steam drum is controlled to be at the centre of the drum.

Discharge valves are provided in the steam mains to release steam to dump condenser in case of pressure rise in the drum. This facility is provided in case the turbine is unable to accept enough steam. In addition to the above mentioned controls, the turbine and the reactor have additional controls for the safe and smooth operation of the plant.

In this thesis, a section of the power plant consisting of the reactor, the primary heat transport system, the heat exchangers, the steam drum and steam valve in the steam mains is simulated. A mathematical model for the above system is formulated, the details of which are given in the next chapter.

CHAPTER 3

NON-LINEAR MODEL

3.1 INTRODUCTION

In the previous chapter, different units of a nuclear power station based on a pressurised heavy water reactor were briefly described. Since the power station has to function as a system, rather than as a collection of individual pieces of equipments, it is essential that the system behaviour as a whole be reasonably well described and understood. This is required in order to establish the necessary control strategies so that the performance of the system meets the specifications imposed from operational, safety and other consideration. In other words, the aim is to design a system i.e., the process plus control, that will do a prescribed job in the 'best' possible way, the best in one case may be determined by economics and in another by transient responses to a set of expected disturbances. In this chapter, the dynamic behaviour of the pressurised heavy water reactor is studied and a mathematical model is obtained as first step towards the design of a control system.

3.2 NEED FOR A MODEL

The first step towards achieving this objective is to completely understand the process being controlled[7]. This begins with a phenomenological description of the process by following the nuclear, physical and chemical changes that

occur along the process path and ends when the time dependent functional relations between the input (controlling) and output (controlled) variables are obtained. This results in a mathematical description of the system, the solution of which predicts the dynamic behaviour of the system to various disturbances. Unfortunately, an accurate mathematical description of the entire system is quite difficult, if not impossible to achieve, since the system is complex and contains numerous variables which are unwieldly to manipulate. Nonlinearities and uncertainities in certain physical phenomena aggravate the complexity of the problem.

To overcome the above difficulty, it is necessary to make some simplifying assumptions. These assumptions may be classified under two categories depending on the nature of simplifications;

(a) Certain physical phenomena are too complex to admit an exact mathematical description. Hence some semi-empirical or approximate formulation is required, examples being heat transfer phenomenon in boiling and two phase flow.

(b) The second type of assumptions stem from the need to obtain a solution within a reasonable time for the set of equations which describe the system. Equations for conservation of momentum and energy and for neutron population in the reactor are in general nonlinear partial differential equations, the exact solutions being limited to simplified cases only. Hence

simplifications may be made in the equations themselves by eliminating nonlinearities or by eliminating partial differentiation through appropriate techniques such as finite difference methods. In the present work, these simplifications along with certain additional assumptions are combined such that a set of ordinary differential equations describe the dynamic behaviour of the plant.

3.3 DYNAMIC BEHAVIOUR

For the purposes of developing mathematical expressions which describe the system, the power plant is divided into the following sub-systems:

i) the reactor and the fuel element with the coolant flowing around it,

ii) the heat exchanger which is further divided into the recirculating leg, the boiling leg and the preheat leg,

iii) the steam drum with the natural circulation loop,

iv) the valve at the steam mains, and

v) the primary heat transport system

Following sections describe the formulation of the dynamic behaviour of the above sub-systems.

It has generally been assumed that the coolant in the primary heat transport system is in the liquid phase only. Changes in its density with temperature variations are neglected, except while calculating the primary circuit

pressure. Also for the two-phase (steam-water) flow, specific volume of the mixture v is calculated as

$$v = xv_g + (1-x)\, v_f$$

where

x = quality of the mixture

v_f, v_g = specific volumes of saturated liquid and vapour respectively

The quality x is calculated as the mass fraction of steam in the two-phase mixture.

3.3.1 Reactor

Dynamic behaviour of the reactor can be characterised by neutron population which is dependent on reactivity. Control mechanism is the primary source for providing reactivity variations. Changes in the temperatures of the fuel and of the coolant in the reactor however introduce reactivity, which is known as feed back reactivity. Thus the temperatures of the coolant and the fuel affect indirectly the behaviour of the reactor.

Temperature of the fuel depends on the power at which the reactor is operating and on the temperature of the coolant in the reactor. It can be visualised that the coolant temperature at inlet to the reactor core is determined by the system external to the reactor, i.e., heat exchanger and steam drum. Hence, changes in operating conditions and variations in the

performance of the external system influence the operation of the reactor.

For purposes of control studies, it is adequate to represent the reactor by a simple point model with six groups of delayed neutrons[30]. The point model kinetics equations are

$$\frac{dn(t)}{dt} = \frac{\delta k(t)-\beta}{\ell^*} n(t) + \sum_{i=1}^{6} \lambda_i c_i(t)$$

and $$\frac{dc_i(t)}{dt} = \frac{\beta_i}{\ell^*} n(t) - \lambda_i c_i(t), \quad i = 1,2,\ldots 6$$

where

$n(t)$ = neutron density

$\delta k(t)$ = reactivity

β = total delayed neutron fraction

ℓ^* = neutron life time

λ_i = decay constant of the i^{th} neutron precursor

$c_i(t)$ = concentration of delayed neutron precursor of group i

β_i = delayed neutron fraction of group i

If neutron density and concentration of the precursor are normalised with respect to their initial steady state values (at time t=0), the equations take the form

$$\frac{dn'(t)}{dt} = \frac{\delta k(t)-\beta}{\ell^*} n'(t) + \sum_{i=1}^{6} \frac{\beta_i}{\ell^*} c'_i \tag{3.1}$$

$$\frac{dc'_i(t)}{dt} = \lambda_i \left[n'(t) - c'_i(t)\right], \quad i = 1,2,\ldots 6 \tag{3.2}$$

where n' and c_i' are normalised values, with initial conditions being $c_i' = 1.0$ for $i = 1,2,\ldots6$ and $n' = 1.0$.

The number of neutrons in the core is proportional to the number of fissions occuring and for 3×10^{10} fissions per second, one watt of thermal power is produced. The power output of a reactor is then proportional to the number of neutrons in the core at a given instant of time and so the symbol n' (or n) is used to designate the neutron level, with the implication that a power level is involved.

Heat generated in the fuel due to fissions occurring there is transferred by conduction in the fuel and the clad. Heat transfer from fuel to clad is a mixture of convection and conduction. Coolant flowing around the clad removes the heat by convective transfer.

Conduction of heat in a solid medium has been discussed in the literature[32,33]. With internal heat generation, the temperature distribution in the solid is given by the following equation:

$$\frac{\partial T}{\partial t} = \frac{k}{\rho c} \nabla^2 T + \frac{q}{\rho c}$$

where

ρ = density of the medium

c = specific heat of the medium

k = thermal conductivity of the medium

q = heat energy developed in unit volume and time

∇^2 = Laplace operator

Thermal conductivity k is in general a function of temperature. The above nonlinear partial differential equation is solved by finite difference technique. With the steady state temperature distribution thus obtained, the fuel dynamics is approximated and the average temperature of the fuel is represented as

$$\frac{dT_{af}}{dt} = 107.6\left(\frac{n'}{n'_{100}}\right) - 0.1938\,(T_{af}-T_{ac}) \qquad (3.3)$$

where n'_{100} refers to the full design value normalised with respect to the initial steady value. Appendix A details the calculation of the coefficients from steady state conditions.

Considering the coolant in the channel as a single lumped system represented by the average temperature of the coolant between inlet and outlet of the core and assuming that the heat loss by the coolant tubes to the moderator is negligible, the average temperature of the coolant can be expressed as

$$\dot{T}_{ac} = 5.44\,(T_{af}-T_{ac})(87/2723\,) + 2.30\,(T_{ri}-T_{ac}) \qquad (3.4)$$

The coefficients are calculated based on steady state conditions.

Reactivity $\delta k(t)$ in Eq. (3.1) can be divided into two parts i.e., one due to feed back δk_{fb} and the other due to control mechanism δk_c. δk_{fb} is written as

$$\delta k_{fb}(t) = \alpha_c\,\Delta T_{ac}(t) + \alpha_f\,\Delta T_{af}(t) \qquad (3.5)$$

where

α_c = temperature coefficient of reactivity for the coolant

α_f = temperature coefficient of reactivity for the fuel

and

ΔT_{af}, ΔT_{ac} = changes in fuel and coolant temperatures from their respective steady state values.

3.3.2 Heat Exchanger

For the purpose of analysis, the heat exchanger is divided into three units - recirculating leg, boiling leg and preheat leg and each unit is simulated separately to study its dynamical behaviour. It is also assumed that all the eighty heat exchangers which are connected to 8 steam drums are identical in their performance.

(a) Recirculating leg

Firstly, the recirculating leg is approximated by a set of n lumped sections. Mass and energy balances for the three media viz., the primary coolant, the tube material and the secondary coolant (water-steam mixture) yield the steady state temperature distribution. Based on the steady state distribution, the riser leg is approximated as a single lumped system. Appendix B outlines the calculation of the dynamic behaviour of the single lumped system and a comparison

of the responses obtained for the single lumped system with that of the distributed system approximated by n lumped section.

The recirculating leg can therefore be represented by

$$\dot{T}_2 = 0.43\ (T_1-T_2) - 2.093\ (T_2-T_{w2})$$
$$\dot{T}_{w2} = 3.296\ (T_2-T_{w2}) - 0.15152\text{x}10^{-3}\ \exp(p/225)(T_{w2}-T_{sat})^4$$
$$\dot{h}_r = \{ 20.834\text{x}10^{-3}\ \exp(p/225)(T_{w2}-T_{sat})^4 + w_d(h_f-h_r) \} /(13.1\ \rho_r) \quad (3.6)$$

Similar analyses are carried out for the boiling leg and the preheat leg (Appendix B) and for these legs the dynamic equations can be written as

(b) Boiling leg

$$\dot{T}_3 = 0.577\ (T_2-T_3) - 1.743\ (T_3-T_{w3})$$
$$\dot{T}_{w3} = 2.702\ (T_3-T_{w3}) - 0.1243\text{x}10^{-3}\ \exp(p/225)(T_{w3}-T_{sat})^4$$
$$\dot{h}_b = \{ 12.81\text{x}10^{-3}\ \exp\ (p/225)(T_{w3}-T_{sat})^4 - w_{mi}(h_b-h_p) \} \text{x}0.102/\ \rho_b \quad (3.7)$$

(c) Preheat leg

$$\dot{T}_4 = 1.76\ (T_3-T_4) - 0.3204\ (T_4-T_{w4})$$
$$\dot{T}_{w4} = 0.6558\ (T_4-T_{w4}) - 5.95\ \left(T_{w4} - \frac{T_{sat}+T_{mi}}{2}\right)$$
$$\dot{h}_p = \left\{ 203.5\ \left(T_{w4} - \frac{T_{sat}+T_{mi}}{2}\right) - w_{mi}(h_p-(h_f-1.1(T_{sat}-T_{mi}))) \right\} /\ (1.39\ \rho_f) \quad (3.8)$$

The symbols have the same meaning as those in Appendix B.

3.3.3 Steam Drum and Natural Circulation Loop

There are eight steam drums in the plant. It is assumed that these eight steam drums behave identically i.e., the flow rates into and out of these drums are identical and the liquid is at the same level in all these drums. Following additional assumptions are made:

i) There is no spatial variation of temperature in the drum and the temperatures of liquid and vapour phases are assumed to be equal to the saturation temperature corresponding to the instantaneous pressure.

ii) The steam separation in the drum is perfect.

iii) Bubble formation in the liquid is neglected.

iv) Heat capacities of drum internals and drum material are neglected.

Based on these assumptions, equations for the conservation of mass and energy are manipulated and the dynamic behaviour of the drum pressure and water level can be written as

$$\dot{p} = (A'-B'-C'-D'b_2\dot{h}_r)/(E'+F'b_3+G'b_1) \quad (3.9)$$

where

$$A' = w_s\, v_g\, h_{fg}$$

$$B' = (v_{fg}\, h_b - v_{fg}\, h_f + v_f\, h_{fg})(w_{mi}+V_b\, \rho_b^2\, b_4\, \dot{h}_b)$$

$$C' = (h_r-h_f)\, v_{fg}\, (w_d+V_r\, \rho_r^2\, b_2\, \dot{h}_r)$$

$$D' = v_r\, h_{fg}\, V_r\, \rho_r^2$$

$$E' = M_g\, (\frac{dv_g}{dp})(h_{fg} - v_{fg}\, E'')$$

$$E'' = M_f\, \frac{d}{dp}\, h_f\, / (M_g\, \frac{d}{dp}\, v_g)$$

$$F' = (v_{fg}\, h_b - v_{fg}\, h_f + v_f\, h_{fg})\, V_b\, \rho_b^2$$

$$G' = \{(h_r - h_f)\, v_{fg} + v_f\, h_{fg}\}\ V_r\, \rho_r^2$$

$$b_1 = (h_r - h_f)\, \frac{d}{dp}\, (v_{fg}/h_{fg}) - \frac{v_{fg}}{h_{fg}}\, \frac{dh_f}{dp}$$

$$b_2 = v_{fg}/h_{fg}$$

$$b_3 = (h_b - h_p)\, \frac{d}{dp}\, (v_{fg}/h_{fg}) - \frac{v_{fg}}{h_{fg}}\, \frac{dh_f}{dp}$$

$$b_4 = b_2$$

$$M_g = (\pi d^2/8 - yd)\, \ell\, \rho_g$$

and $M_f = (\pi d^2/8 + yd)\, \ell \rho_f$

$$\dot{y} = (A' + B' - C' - D')/(E' \times F') \qquad (3.10)$$

where

$$A' = w_{mo}\, (h_g - h_b)$$

$$w_{mo} = w_{mi} + V_b \rho_b^2\, (b_4\, \dot{h}_b + b_3\, \dot{p})$$

$$B' = w_r\, (h_g - h_r)$$

$$w_r = w_d + V_r \rho_r^2\, (b_2\, \dot{h}_r + b_1\, \dot{p})$$

$$C' = w_d\, h_{fg}$$

$$D' = ((w_{mo} - w_s) + (w_r - w_d))\, v_g \cdot E''$$

$$E' = h_{fg} - v_{fg}\, E''$$

$$E'' = M_f \frac{dh_f}{dp} / (M_g \frac{dv_g}{dp})$$

$$F' = \rho_f \ell d$$

and M_f, M_g, b_1, b_2, b_3 and b_4 as defined in Eq. (3.9).

Momentum balance for the natural circulation loop consisting of the steam drum, the downcomer and the riser yields the flow rate in the downcomer as

$$\dot{w}_d = [20(\rho_f - \rho_r) - \frac{2.21}{\rho_f} w_d^2 - \frac{w_r^2}{\rho_r} \{ 1.036 + 0.274 \{ (1 + 2400 (\frac{x_r}{p})^{.96} \}^{\frac{\rho_r}{\rho_f}} \} + 0.95z] / 5.51 \qquad (3.11)$$

where

$$z = 2V_r \rho_r^3 (b_1 \dot{p} + b_2 \dot{h}_r)^2 - 2V_r \rho_r^2 a_1(\dot{h}_r - a_2\dot{p})\, \dot{p}$$

$$a_1 = \frac{d}{dp} (v_{fg}/h_{fg})$$

$$a_2 = \frac{d}{dp} h_f$$

and $w_r = w_d + \rho_r^2 V_r (b_1 \dot{p} + b_2 \dot{h}_r)$

where b_1 and b_2 are given in Eq. (3.9).

Details of the formulation are provided in Appendix C.

3.3.4 Thermodynamic Properties of Steam

Over the design range of pressure from 500 psia to 700 psia, the specific volume of saturated liquid is assumed to be a constant whereas other thermodynamic properties are

assumed to vary linearly with pressure. The functional relationships are provided in Appendix C.

3.3.5 Steam Flow to Turbine

Sonic flow through the valve is assumed as a convenient way of eliminating the consideration of downstream pressure effects[34]. The flow through the valve is then dependent upon the valve area and the upstream pressure only and the steam mass flow rate is written as

$$w_s = \text{constant} \times a_s (p/v_g)^{\frac{1}{2}}$$

where a_s = area of steam valve opening.

Normalising the flow with respect to the rated opening of the valve, the steam flow can be written as

$$\frac{w_s}{w_{100}} = w'_s = a_s \left[\frac{p}{p_{100}} \frac{v_{g,100}}{v_g} \right]^{\frac{1}{2}} \qquad (3.12)$$

where a_s = normalised flow area, w'_s = normalised steam mass flow rate, and p_{100}, $v_{g,100}$ are pressure and specific volume of saturated vapour at hundred percent load.

3.3.6 Pressure of Primary Heat Transport System

The primary heat transport system is of constant volume with coolant in the liquid phase. The pressure of primary coolant in the outlet headers of the reactor is assumed to depend on the mass of coolant in the entire circuit and the average temperature in the circuit. The

average temperature is calculated as a weighted value of the temperatures in the reactor and the heat exchanger[35] and is given by

$$T_{pav} = 0.2126\ T_{ro} + 0.2069\ (T_{ri}+T_1) + 0.0862(T_2+T_3) + 0.2010\ T_4 \qquad (3.13)$$

With V_D being the volume of the primary circuit, the pressure of coolant P is given by[35]

$$P = \left[\frac{B'}{(v_D - A')} - C' \right] \times 14.22 \qquad (3.14)$$

and $v_D = V_D/M_D$

where P = pressure in psia

v_D = specific volume (in cm^3/gm)

V_D = volume of the coolant circuit

$= 4.28 \times 10^7\ cm^3$

M_D = mass of the primary coolant in the circuit (in gms)

$A' = 0.28597\ T_k - 0.6909$

$B' = 8594.06 - 111367.06/T_k + 365541.88/T_k^2$

$C' = 16792.78 - 242883.66/T_k + 865909.5/T_k^2$

and $T_k = 0.01\ [(T_{PAV}-32)/1.8 + 273)]$

It is assumed that the mass of the coolant M_D can be controlled by a feed or bleed valve such that

$$\frac{dM_D}{dt} = \text{constant} \times a_D \qquad (3.15)$$

where a_D = valve area normalised with respect to the total valve area

In the above equation a_D is expressed as a percentage and M_D in kilograms. The constant is taken as 0.117 which corresponds to a mass flow of 11.7 kg/sec (volume flow of 817 litres per minute) at rated valve opening[31].

The feedwater flow to the inlet of the preheater leg is controlled by a valve. It is assumed that the flow through the valve is dependent on the valve area and is independent of the drum pressure. Therefore the flow rate is written as

$w_{mi} = a_1 \times$ area of valve opening, where a_1 is a constant.

Normalising the flow with respect to full flow at the rated opening of the valve, the feedwater flow can be written as

$$w_{mi} = a_m \times w_{m,100}$$

where $w_{m,100}$ is the rated flow at full load

and a_m is the area of the valve opening expressed as a fraction of rated area at full load

Further, the transport delay between the reactor and the heat exchanger can be expressed as

$$T_{ri}(t) = T_4\,(t-\tau_1)$$

$$T_1(t) = T_{ro}(t-\tau_2)$$

and $$T_{ro}(t) = 2\,T_{ac}(t) - T_{ri}(t) \qquad (3.16)$$

Here T_{ri}, T_{ro} refer to the temperatures of the coolant at inlet to and outlet of the reactor. T_1 and T_4 refer to the temperatures of the coolant at inlet and outlet of the heat exchanger.

Equations (3.1) to (3.16) can be arranged in the form

$$\dot{\underline{x}} = f(\underline{x}(t), \underline{u}(t), \underline{w}(t), \underline{x}(t-\tau_1), \underline{x}(t-\tau_2))$$

and $$\underline{y} = g(\underline{x}(t), \underline{u}(t), \underline{w}(t)) \tag{3.17}$$

where

$$\underline{x}^T = (n', c_i, i=1,2,\ldots,6, T_{af}, T_{ac}, T_2, T_{w2}, h_r, T_3, T_{w3}, h_b, T_4, T_{w4}, h_p, w_d, p, y, M_D)$$

$$\underline{u}^T = (\delta k_c, a_m, a_D)$$

$$\underline{w}^T = (a_s, T_{mi})$$

and $$\underline{y}^T = (p, T_{ac}, P, y)$$

Here $\underline{x}$ refers to the variables which characterise the system,

$\underline{u}$ refers to inputs which are controllable (or which are used for controlling the performance of system)

$\underline{w}$ refers to inputs which are uncontrollable

and $\underline{y}$ refers to certain outputs of the system.

Equation 3.17 then represents the dynamical behaviour of the plant.

3.4 STEADY STATE CONDITIONS

In order to study the dynamic behaviour of the reactor around full load conditions, steady state values of $\underline{x}$ and $\underline{y}$

at full load conditions namely $\underline{x}^o$ and $\underline{y}^o$ respectively, are needed. These are obtained by the following procedure: If the values of $\underline{u}$ and $\underline{w}$ at full power operation of the plant namely $\underline{u}^o$ and $\underline{w}^o$ respectively, are specified, then Eq. (3.17) can be integrated till the steady state is reached. This is equivalent to obtaining the transient response of the plant for specified $\underline{u}$ and $\underline{w}$ from some initial $\underline{x}$. In this study the initial value of $\underline{x}$ is formed from the values obtained from the steady state studies of the various sub-systems such as the riser, boiling and preheat legs of the heat exchanger and the natural circulation loop. The values of $\underline{u}^o$, $\underline{w}^o$ and other parameters which characterise the full power operation of the reactor are given below.

i) The reactor is operating at full power, i.e., n'_{100} in Eq. (3.3) is taken as 1.0.

ii) Since ΔT_{ac} and ΔT_{af} at steady state are to be zero, δk_{fb} in Eq. (3.5) is taken as zero.

iii) $\delta k_c = 0$

iv) To maintain the water level at the centre of the drum, feed water flow rate w_{mi} is taken to be equal to the steam flow rate w_s. Since the flow rates of steam and feed water are equal at full power, this leads to $a_m = a_s$.

v) a_D is used to adjust M_D so that P is held at 1250 psia.

vi) Feed water inlet temperature T_{mi} is taken as 340^oF.

vii) Steam valve opening is at the rated design value, i.e., $a_s=1.0$.

With these values of $\underline{u}^o$ and $\underline{w}^o$, $\underline{x}^o$ and $\underline{y}^o$ are obtained as

$$
\begin{aligned}
(\underline{x}^o)^T &= (1.0,\ 1.0,\ 1.0,\ 1.0,\ 1.0,\ 1.0,\ 1.0, \\
&\quad 1075.6,\ 520.7,\ 503.4,\ 491.6,\ 550.65, \\
&\quad 492.3,\ 488.6,\ 592.3,\ 480.7,\ 416.9, \\
&\quad 474.7,\ 61.6,\ 568.9,\ 0.0,\ 0.363\text{x}10^5) \qquad (3.19) \\
(\underline{y}^o)^T &= (568.9,\ 520.7,\ 1250.3,\ 0.0)
\end{aligned}
$$

Also $\underline{u}^o$ and $\underline{w}^o$ can be written as

$$(\underline{u}^o)^T = (0.0,\ 1.0,\ 0.0)$$

and $$(\underline{w}^o)^T = (1.0,\ 340.0)$$

These values of $\underline{x}^o$, $\underline{y}^o$, $\underline{u}^o$ and $\underline{w}^o$ represent the steady state conditions of the reactor at full power operation. Further these values give a steam flow rate of 8.714 lb/sec. per heat exchanger. This corresponds to a total steam production of $2.5096\text{x}10^6$ lb/hr. at 569 psia.

The major advantage of obtaining the steady state values by a transient response analysis, as compared with procedures computing the steady state conditions by purely time-independent relationship such as minimisation of a function, is as follows[36]. There is a need for the verification of dynamic equations and this can be achieved by bringing the plant to steady state through a dynamic analysis. Obtaining a satisfactory steady state check is an indication of the

absence of gross errors in the model.

3.5 TRANSIENT RESPONSES

Effect of various inputs, both controllable and uncontrollable, on the system which is initially at steady full load operating conditions is now studied. A number of inputs such as change in steam valve area a_s, change in the inlet temperature of the feed water T_{mi}, oscillations in input reactivity δk_c, change in the area of feedwater valve a_m are applied and the responses are obtained. However for the sake of brevity, the responses due to two inputs namely change in a_s and change in T_{mi} are presented in this section.

3.5.1 Response for a Step Change in Steam Valve Opening:

Figure 3.1(a-g) shows the response for a -5 percent step change in valve area.

The step change causes a sudden decrease (Fig.3.1e) in the steam mass flow rate (w'_s), which initiates the trend of pressure rise in the drum (Fig.3.1d). Increase in pressure leads to an increase in saturation temperature. This, with a constant temperature for the incoming primary coolant, results in decreased heat transfer and reduced steam production. Further, the reduced energy transfer from the primary coolant leads to a rise in the temperature of the primary coolant leaving the heat exchanger. However, this effect is felt

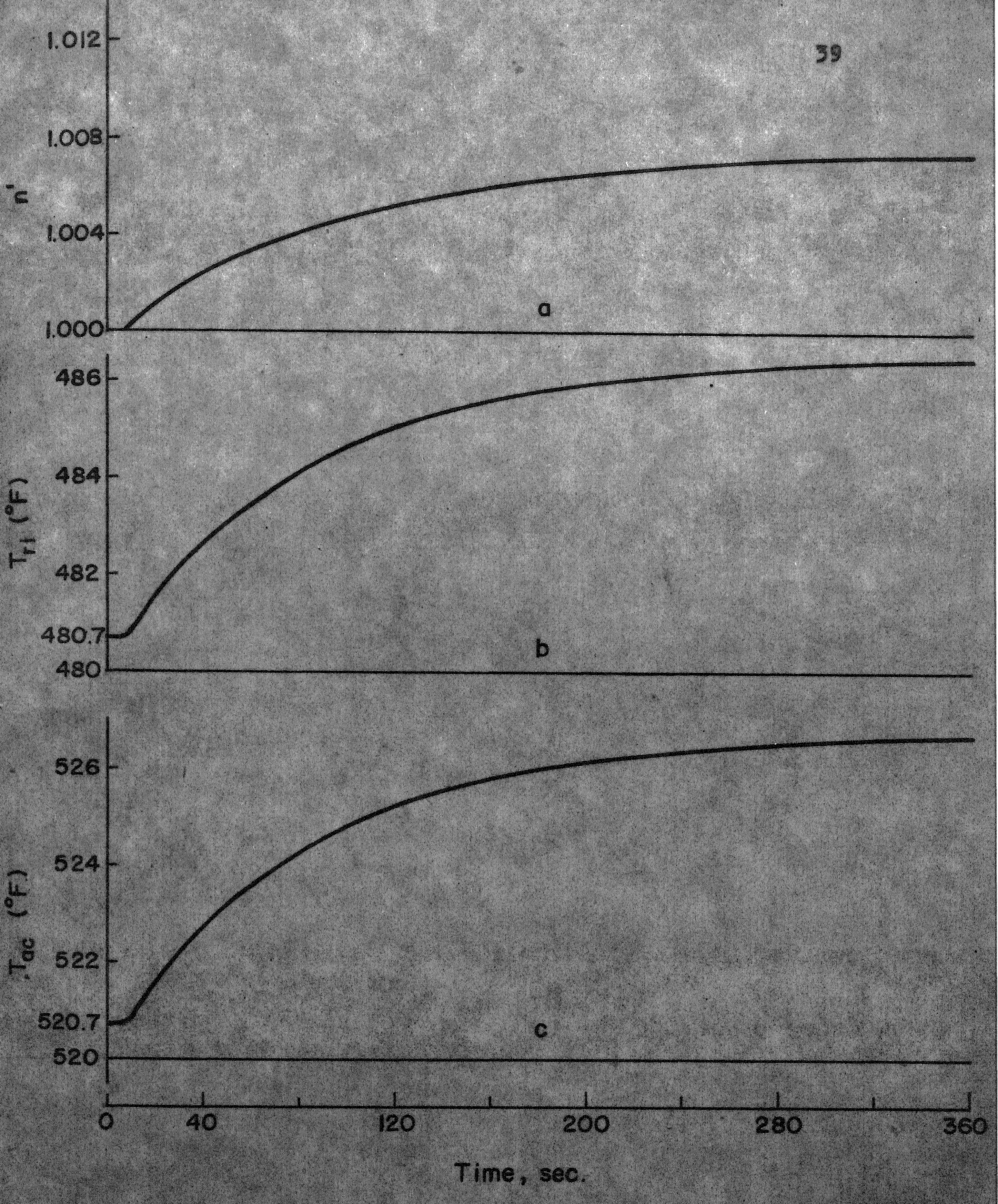


Fig. 3.1 Uncontrolled Response Of Nonlinear Model For a −5% Step In Steam Valve Area

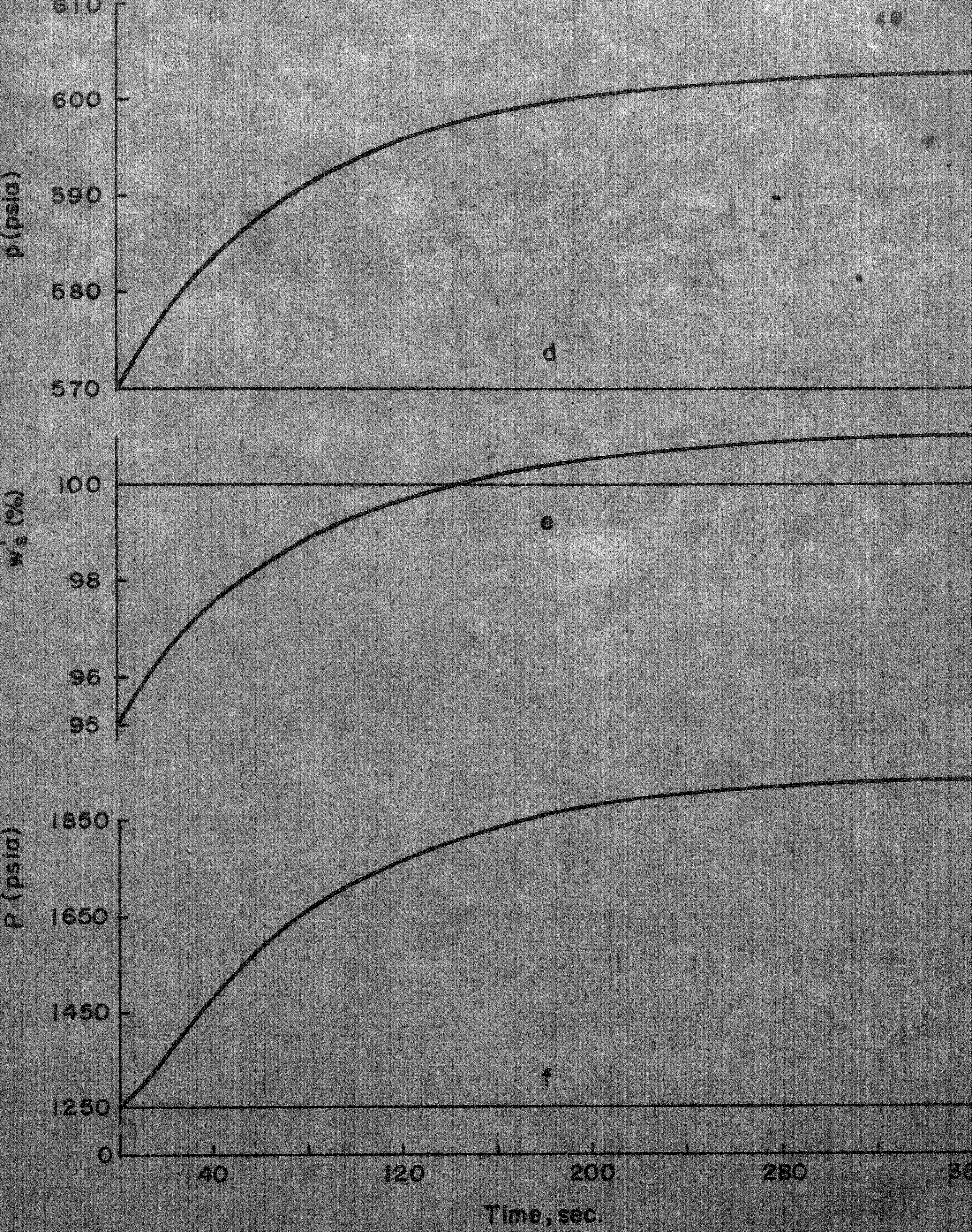

610
600
590
580
570
p (psia)
d
100
98
96
95
w_s' (%)
e
1850
1650
1450
1250
0
P (psia)
f
40
120
200
280
36
Time, sec.

Fig. 3.1 (Contd.)

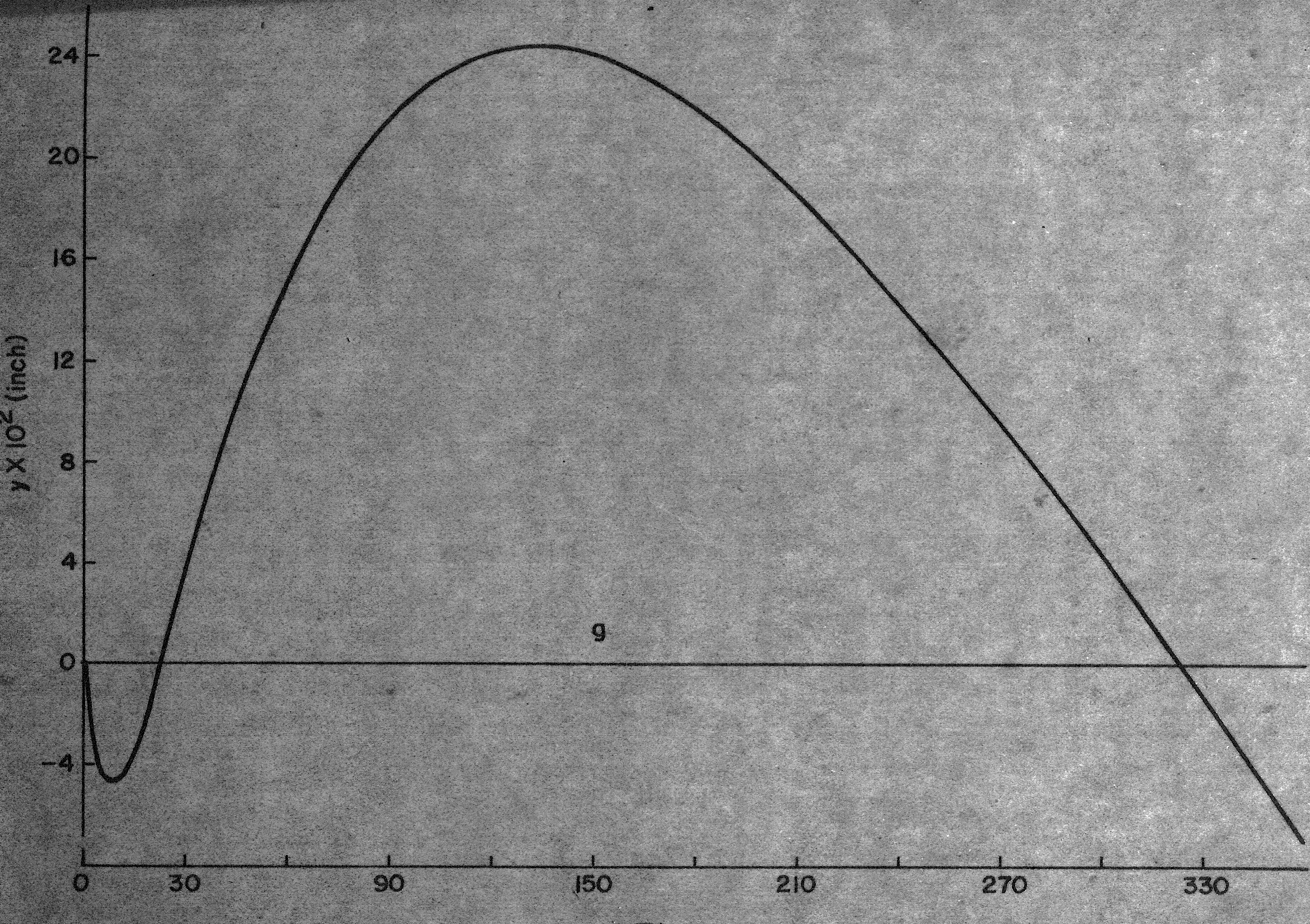


Fig. 3.1 (Contd.)

at the reactor only after a period equivalent to the transport delay. Reduced steam production coupled with an increase in steam outflow which is due to the pressure rise, reduces the rate of pressure rise with the pressure eventually attaining a steady value (Fig.3.1d).

The steam flow is determined by the flow area and the steam pressure. The sudden decrease in the steam mass flow rate, following the closure of valve, cannot be maintained as pressure increases. Hence the steam flow gradually rises with the pressure, reaching a final steady value which is slightly more than the original equilibrium value (Fig.3.1e).

The drum level is directly related to the mass of water in the drum which in turn depends on the flow rates entering and leaving the drum. With an increase in pressure, the quality of steam in the flow from boiling and riser legs decreases. This increases the amount of liquid held in the legs leading to a decrease in the mass flow rates of water leaving them. This accounts for the initial drop in drum level (Fig.3.1g). However, the decreased steam flow rate from drum with a constant feed water flow to the preheat leg causes the drum level to rise in a short time. The rise continues for 130 seconds until the steam flow rate is more than the feed water flow rate, after which the drum level drops steadily.

The increase in drum pressure leads to a rise in the temperature of the primary coolant entering the reactor. With the external reactivity input being held at equilibrium level, this increase leads to a rise in the average temperature of the coolant (Fig.3.1c) and a consequent rise in the temperature of the coolant leaving the reactor. This results in a small positive reactivity which raises the powerlevel of the reactor. However, the increase in fuel temperature due to the increase in temperature of the coolant and in the power level introduces negative reactivity to compensate the excess reactivity, and the reactor power reaches a steady value (Fig.3.1a). The average temperature of the primary coolant is higher than the equilibrium steady state value. Since the mass of coolant is unaltered (the feed/bleed valves are not controlled), the primary circuit pressure rises enormously (Fig.3.1f) before attaining a steady value.

3.5.2 Response for a Step Change in Feed Water Temperature:

Figure 3.2 shows the response for a-15 ^{O}F step change in the temperature of feed water entering the steam generator.

The step change in feed water temperature reduces the average temperature in the preheat leg, leading to an increase in energy transfer to feed water and a consequent decrease in the temperature of the primary coolant leaving the steam generator. The increased energy transfer to feed water causes

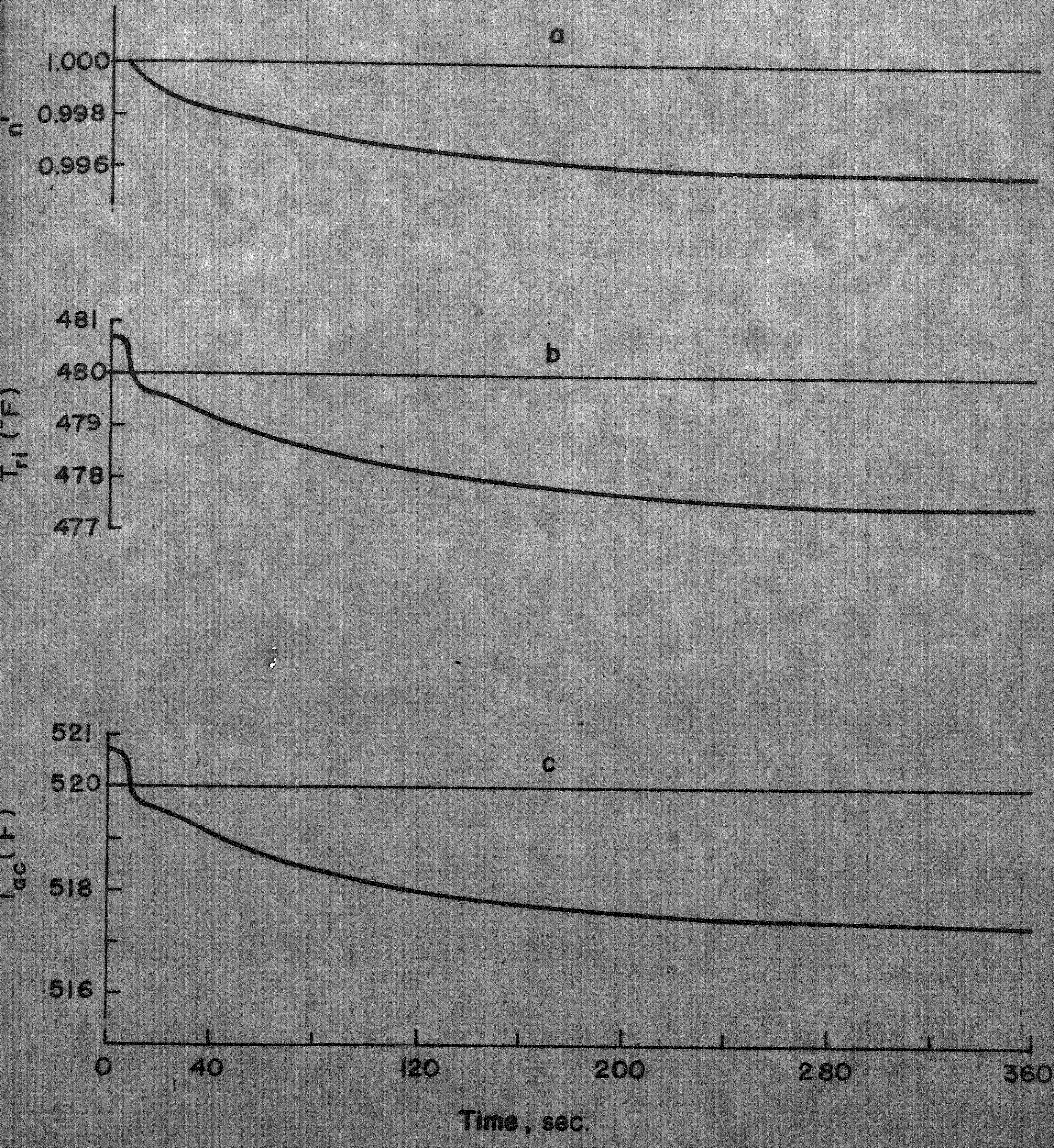
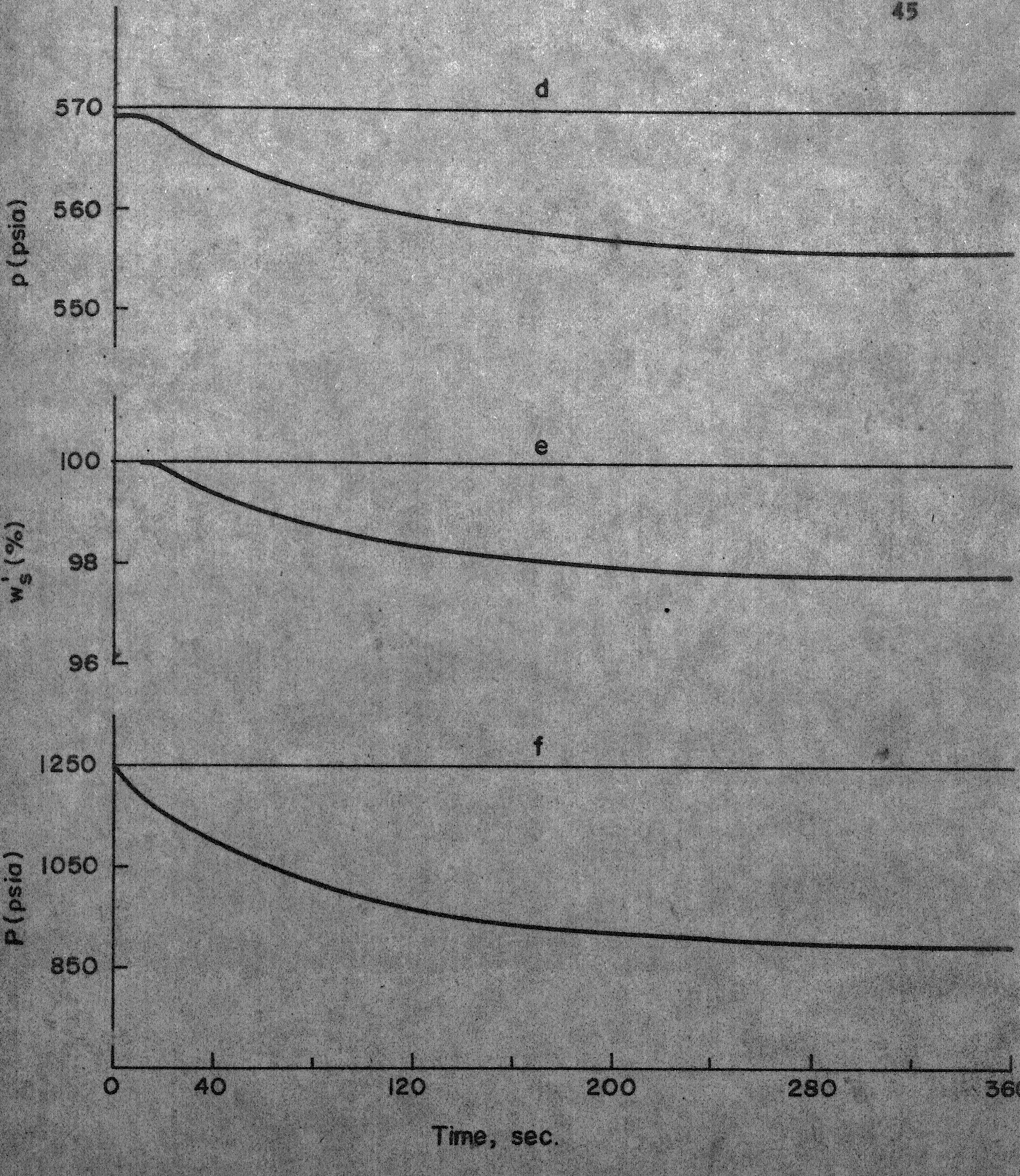


Fig. 3.2 Uncontrolled Response Of Nonlinear Model For a -15°F Step In Feed Water Inlet Temperature



d
570
560
550
p (psia)
e
100
98
96
w_s' (%)
f
1250
1050
850
P (psia)
0
40
120
200
280
360
Time, sec.

Fig. 3.2 (Contd.)

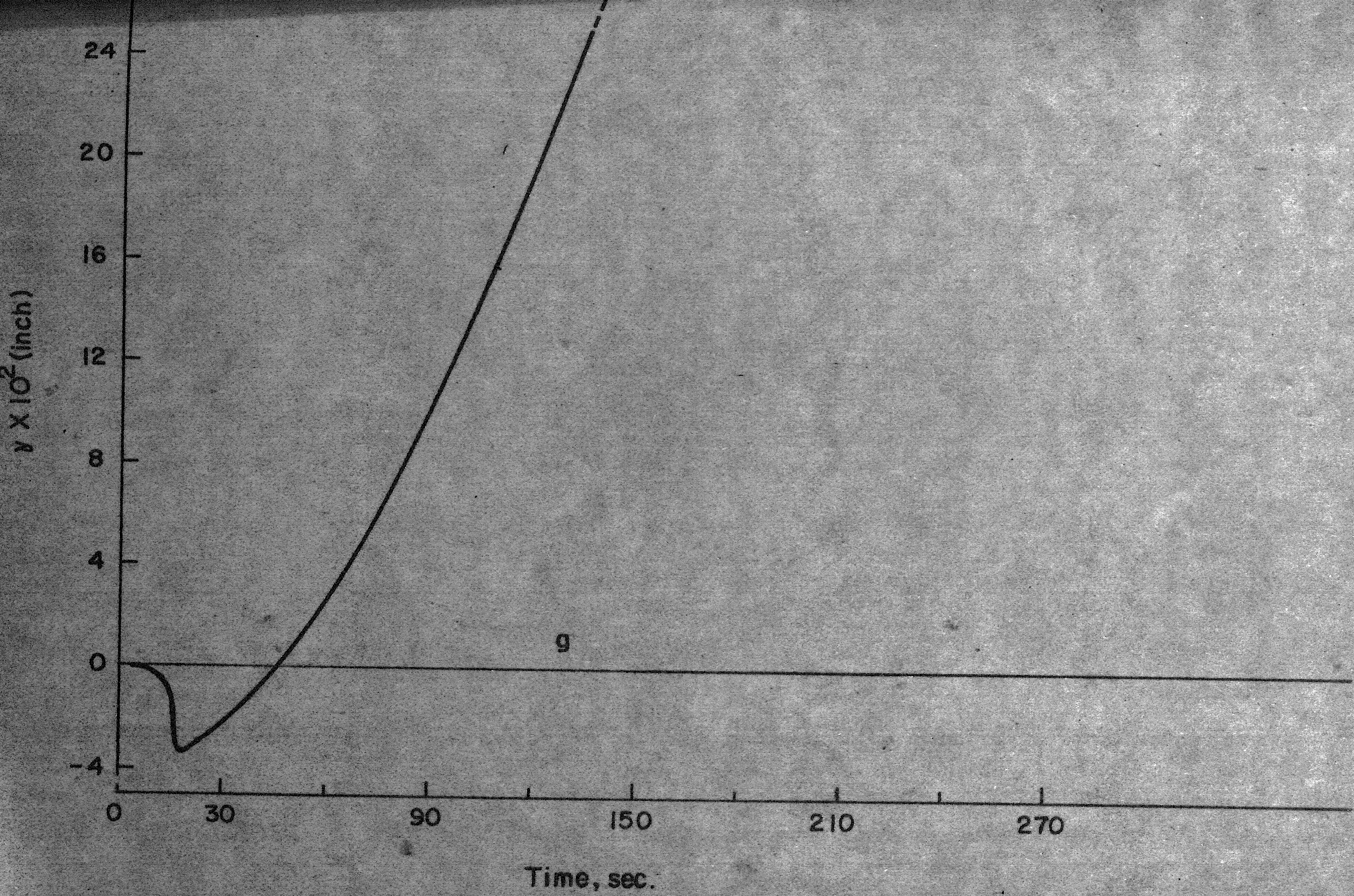


Fig. 3.2 (Contd.)

the enthalpy at the outlet of the preheat leg to remain constant or increase slightly. Hence immediately after the application of the step there are no changes in drum pressure and in drum level (Fig.3.2d and 3.2g). The drop in the outlet temperature of primary coolant at the steam generator is felt at the reactor after a time lag equivalent to the transport delay (Fig.3.2b). With the external reactivity input being held at its equilibrium level, the reactor power is constant at its equilibrium value. This, coupled with the reduced inlet temperature of the coolant to the reactor, results in a decrease of the outlet temperature of the coolant. Hence after a time, which is equivalent to the sum of transport delays between the reactor and the steam generator, the steam generator is supplied with the coolant at a lower temperature. This causes a reduction in steam production leading to a decrease in the drum pressure and the steam flow (Fig.3.2d and 3.2e). Further, a reduction in steam production increases the density of steamwater mixture leaving the recirculating and boiling legs and the mass of liquid contained in these legs. This leads to a decrease in the flow of liquid to the steam drum. This in turn is reflected (Fig.3.2g) as a decrease in the liquid level. With the feed water mass flow at its equilibrium value, the reduced steam flow soon explains the rise in the liquid level.

The decrease in the average temperature of the coolant results in a small negative reactivity which reduces the power level of the reactor. However the decrease in fuel temperature due to the decrease in the temperature of the coolant and in the power level introduces positive reactivity to compensate the negative reactivity, and the reactor power reaches a steady value (Fig.3.2a). The average temperature of the primary coolant is lower than the equilibrium steady state value. Since the mass of coolant is unaltered (the feed/bleed valves are not controlled), the primary circuit pressure decreases enormously before reaching a steady value (Fig.3.2f). (Eq.3.14 is assumed to hold good, i.e., the heavy water is in liquid phase only).

3.6 CONCLUSIONS

These responses appear realistic and the trends compare with the expected behaviour of the plant. The response of the steam pressure and water level in the drum agree qualitatively with the results obtained in field tests of a boiler[12,13] and with other theoretical results on drum type natural circulation boilers[7,8,37]. However the proposed model can only be validated by comparison with the uncontrolled responses of the plant. Unfortunately such test data are not available. Since the nature of the responses agree with physical reasoning, there is added confidence in the model.

In this chapter, a nonlinear model for the pressurised heavy water reactor is formulated. Uncontrolled responses of the system for various disturbances are obtained and are discussed. In order to design a control system for the reactor, the nonlinear model is linearised around operating conditions. This results in a linear time invariant system which is valid for only small deviations from the operating point. This linear model is developed in the next chapter.

CHAPTER 4

LINEAR MODEL

4.1 INTRODUCTION

Many of the differential equations developed in the previous chapter are nonlinear. For small perturbations about a steady operating point, these equations can be linearised to provide a set of linear first order ordinary differential equations. Such a system will facilitate the use of well developed control techniques in designing feedback controls. Hence in this chapter, the plant is formulated as a linear system.

4.2 MODIFICATIONS OF THE NONLINEAR MODEL

The nonlinear model of the plant developed in Chapter 3 is modified as given below.

A simplification is made in the neutron kinetics. Instead of six groups of delayed neutrons, only one average precursor group will be used in subsequent analysis. The decay constant of the one group precursor is then calculated as

$$\lambda = \frac{\beta}{\sum_{i=1}^{6} [\beta_i/\lambda_i]}$$

$$= 0.0708 \text{ sec}^{-1}$$

Transport delays between the reactor and the steam generator are represented by a first order approximation such that

$$\dot{T}_{ri} = (T_4 - T_{ri})/\tau_1 \qquad (4.1)$$

$$\dot{T}_1 = (T_{ro} - T_1)/\tau_2$$

Since $T_{ro} = 2\, T_{ac} - T_{ri}$,

$$\dot{T}_1 = (2\, T_{ac} - T_{ri} - T_1)/\tau_2 \qquad (4.2)$$

Here T_{ri}, T_{ro} refer to the temperature of the coolant at the inlet and at outlet headers of the reactor respectively; T_1, T_4 refer to the temperature of the coolant at the inlet and at the outlet of the steam generator respectively.

These Eqs.(4.1) and (4.2) along with the equations developed in the previous chapter are considered to represent the system and are linearised around the steady state values.

4.3 LINEARISATION PROCEDURE[37]

Considering the general equation of the form

$f(x_1, x_2, \ldots x_n; u_1, u_2, \ldots u_m, \dot{x}_1, \dot{x}_2 \ldots \dot{x}_n) = 0$

and perturbing about the steady state values to eliminate the nonlinearities, gives

$$\frac{\partial f}{\partial x_1}\Delta x_1 + \frac{\partial f}{\partial x_2}\Delta x_2 \ldots + \frac{\partial f}{\partial x_n}\Delta x_n + \frac{\partial f}{\partial u_1}\Delta u_1 + \ldots \frac{\partial f}{\partial u_m}\Delta u_m$$
$$+ \frac{\partial f}{\partial \dot{x}_1}\Delta \dot{x}_1 \ldots + \frac{\partial f}{\partial \dot{x}_n}\Delta \dot{x}_n \equiv 0$$

For small perturbations, $\Delta\frac{dx_1}{dt}$, ... $\Delta\frac{dx_n}{dt}$ may be replaced by $\frac{d}{dt}\Delta x_1, \ldots \frac{d}{dt}\Delta x_n$ and the partial differentials $\frac{\partial f}{\partial x_1}, \ldots \frac{\partial f}{\partial u_1}$, ..., $\frac{\partial f}{\partial \dot{x}_1}$ ..., that form the coefficients of the perturbed

variables are evaluated at the initial steady state values about which the dynamic behaviour of the system is to be analysed. The unknowns in the equation are the perturbed values $\Delta x_1, \ldots \Delta x_n$, $\Delta u_1, \ldots \Delta u_m$, $\Delta \dot{x}_1, \ldots \Delta \dot{x}_n$.

4.4 LINEAR MODEL

For example, considering the neutron kinetics equation one average precursor,

$$\frac{dn'}{dt} = \frac{\delta k-\beta}{\ell^*} n' + \frac{\beta}{\ell^*} c'$$

where $\delta k = \alpha_f (T_{af} - T^o_{af}) + \alpha_c (T_{ac} - T^o_{ac}) + (\delta k_c - \delta k^o_c)$

Initial steady state values are:

$$T_{af} = T^o_{af}$$
$$T_{ac} = T^o_{ac}$$
$$\delta k_c = \delta k^o_c = 0.0$$
$$n' = 1.0$$
$$c' = 1.0$$

Pert urbing the equation by the procedure given in Section 4.3, it can be written as

$$\frac{d}{dt}(\Delta n') = \left(\frac{\delta k-\beta}{\ell^*}\right)_o \Delta n' + \left(\frac{\beta}{\ell^*}\right)_o \Delta c' + \left(\frac{n'}{\ell^*}\right)_o \alpha_f \Delta T_{af} + \left(\frac{n'}{\ell^*}\right)_o \cdot \alpha_c \Delta T_{ac} + \left(\frac{n'}{\ell^*}\right)_o \delta k_c$$

where suffix o indicates that the terms are evaluated based on steady state values and regarded as constants in further analysis. Hence this can be written as

$$\Delta \dot{n}' = a_1 \Delta n' + a_2 \Delta c' + a_3 \Delta T_{af} + a_4 \Delta T_{ac} + a_5 \delta k_c$$

This is now a linear equation in the perturbed variables $\Delta n'$, $\Delta c'$, ΔT_{af}, ΔT_{ac}, δk_c.

In section 4.2, modifications are proposed to the nonlinear model developed in Chapter 3. These modifications are incorporated in the dynamics of the plant namely Eq.(3.17) and the resulting equations are linearised by the above procedure. A set of linear ordinary differential equations is thus obtained and this can be expressed as

$$\underline{\dot{x}} = \underline{A}\ \underline{x} + \underline{B}_1\underline{u} + \underline{B}_2\ \underline{w} \tag{4.3}$$

where $\underline{x}$, $\underline{u}$, $\underline{w}$ represent, hereafter, the perturbed values i.e., $\Delta\underline{x}$, $\Delta\underline{u}$, $\Delta\underline{w}$ respectively, from their steady state values. $\underline{x}$, $\underline{u}$ and $\underline{w}$ are vectors of 19, 3 and 2 variables respectively and $\underline{A}$, $\underline{B}_1$, $\underline{B}_2$ are matrices of orders (19x19), (19x3) and (19x2) respectively. Thus for the power plant,

$$\underline{x}^T = (n, c, T_{af}, T_{ac}, T_2, T_{w2}, h_r, T_3, T_{w3}, h_b, T_4,$$
$$T_{w4}, h_p, w_d, p, y, T_1, T_{ri}, M_D)$$
$$\underline{u}^T = (\delta k_c, a_m, a_D)$$

and $\quad \underline{w}^T = (a_s, T_{mi})$

where n = changes in the power level expressed as a percentage of full power

y = changes in water level expressed in the unit of 0.01 inch

δk_c = reactivity inputs in cents, with 100 cents being equal to a reactivity of 0.0064.

a_m, a_D, a_s are changes in the area of the feed water control valve, primary circuit pressure control valve area and steam valve area respectively and are expressed as percentages of full opening (positive and negative values for a_D refer to the feed and the bleed valves respectively) and the other symbols having the same meaning as in Chapter 3.

Further, Eqs. (3.14) and (3.12) for the calculation of the primary circuit pressure and the steam mass flow rate can be linearised in a similar manner and this yields

$$P = 2.214\ x_{19} + 49.14\ x_4 - 0.64\ x_{18} + 23.93\ x_{17} + 9.963\ x_5 + 9.963\ x_8 + 23.24\ x_{11} \tag{4.4}$$

and

$$w_s' = 0.1776\ x_{15} + w_1 \tag{4.5}$$

where x_i, u_i, w_i refer to the i^{th} variable of the vector $\underline{x}$, $\underline{u}$ and $\underline{w}$ respectively.

From Eq. (4.4),

$$\dot{P} = 2.214\ \dot{x}_{19} + 49.14\ \dot{x}_4 - 0.64\ \dot{x}_{18} + 23.93\ \dot{x}_{17} + 9.963\ \dot{x}_5 + 9.963\ \dot{x}_8 + 23.24\ \dot{x}_{11}$$

Substituting for the derivatives of the state variables x_i from Eq. (4.3),

$$\dot{P} = 10.8\ x_3 - 156.0\ x_4 - 25.9\ x_5 + 27.7\ x_6 + 23.7\ x_8 + 23.1\ x_9 - 67.0\ x_{11} + 9.89\ x_{12} + 3.33\ x_{17} + 151.0\ x_{18} + 0.262\ u_3 \tag{4.6}$$

Eq. (4.6) is then used in place of the first order differential equation for x_{19} in Eq.(4.3) and Eq.(4.3) can be written as

$$\dot{\underline{x}} = A\underline{x} + B_1\underline{u} + B_2\underline{w} \qquad (4.7)$$

with the last variable x_{19} now being the primary circuit pressure P.

The matrices $\underline{A}$, $\underline{B}_1$ and $\underline{B}_2$ for the above system are provided in Tables 4.1, 4.2 and 4.3.

4.5 TRANSIENT RESPONSES

Uncontrolled dynamic behaviour of the system represented by Eqs.(4.7) and (4.5) is obtained for two inputs, which are same as those considered in Chapter 3, namely a -5 percent step in the area of the steam valve w_1 and a -15 degree step in the feed water temperature w_2, with the initial conditions being $\underline{x}$=0 and $\underline{u}$=0 at time t=0. These responses are shown in Fig.4.1 and 4.2. These transient responses are in close agreement with those obtained for the nonlinear model (in Chapter 3), thereby validating the linearisation procedure. The difference between the responses shown in Fig.4.2b,c and Fig.3.2b,c is mainly due to the first order approximation of the transport delays between the reactor and the steam generator. A higher order approximation will improve the agreement during the early part of the transients. But this will increase the computational efforts needed for control studies. In view of this

Table 4.2

Matrix B_1 (19x3 Matrix)

$\underline{B}_1$ has 7 non-zero elements and 50 zero elements. The non-zero elements are:

$B_1(\ 1,\ 1) = \ \ 0.94E+01$

$B_1(10,\ 2) = -0.17E+00$

$B_1(12,\ 2) = -0.33E+00$

$B_1(13,\ 2) = -0.42E-01$

$B_1(15,\ 2) = \ \ 0.73E-02$

$B_1(16,\ 2) = \ \ 0.30E+00$

$B_1(19,\ 3) = \ \ 0.26E+00$

Table 4.3

Matrix B_2 (19x2 Matrix)

$\underline{B}_2$ has 4 non-zero elements and 34 zero elements. The non-zero elements are:

$B_2(15,\ 1) = -0.18E+00$

$B_2(16,\ 1) = \ \ 0.55E+00$

$B_2(12,\ 2) = \ \ 0.29E+01$

$B_2(13,\ 2) = -0.13E+01$

In the tables given above A(I,J) represents $a_{i,j}$ of matrix A. Also, the symbol E stands for the power of 10. For example A(1,1) = -0.94E+01 indicates $a_{1,1}=0.94x10^{+1}$.

the first order approximation is considered as acceptable. Hence the system represented by Eq.(4.7) can be utilised for studying the dynamical behaviour for small perturbations. In the subsequent chapters viz. Chapters 5 and 6, the linear system is studied to develop feed back controls based on the optimal control theory.

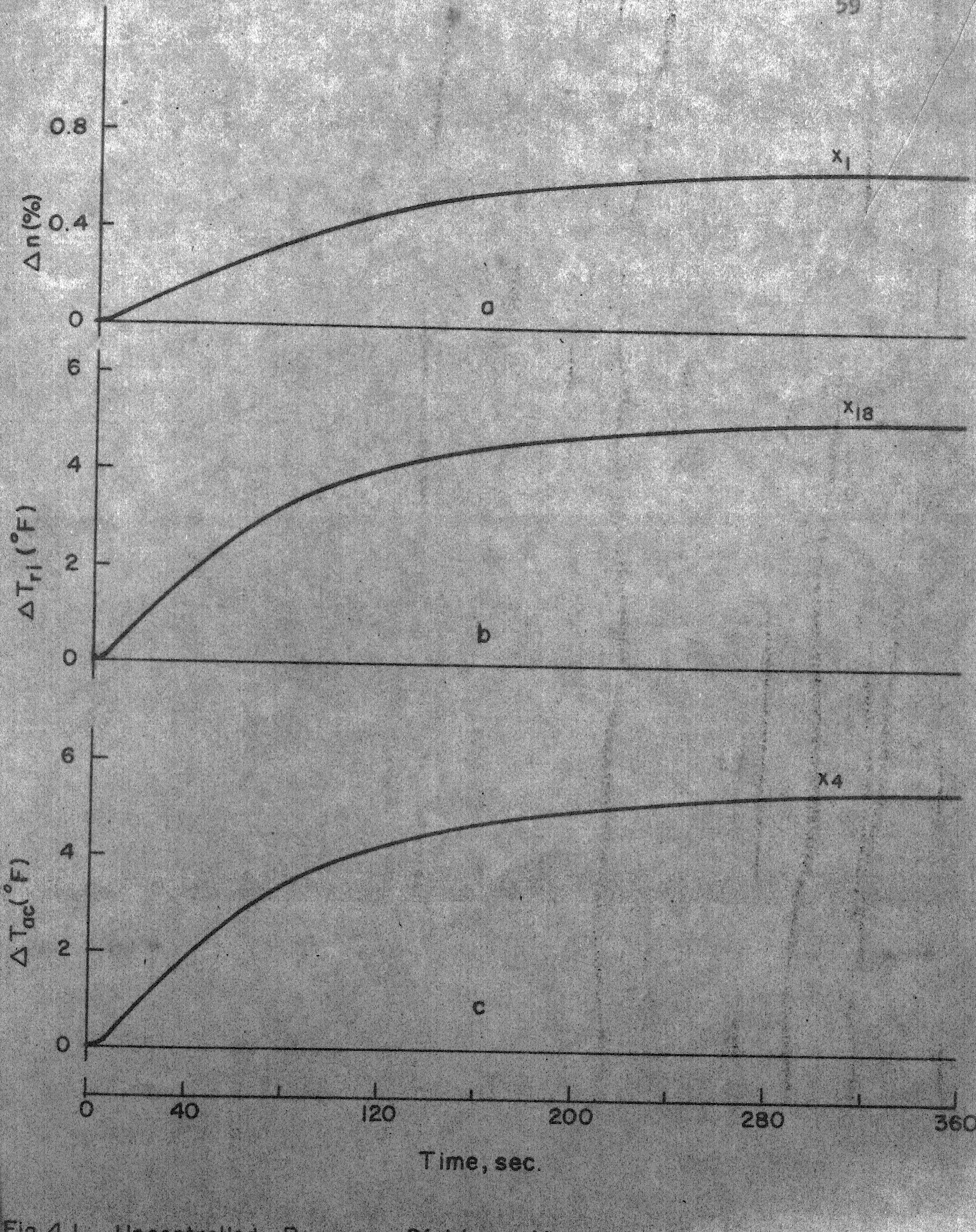
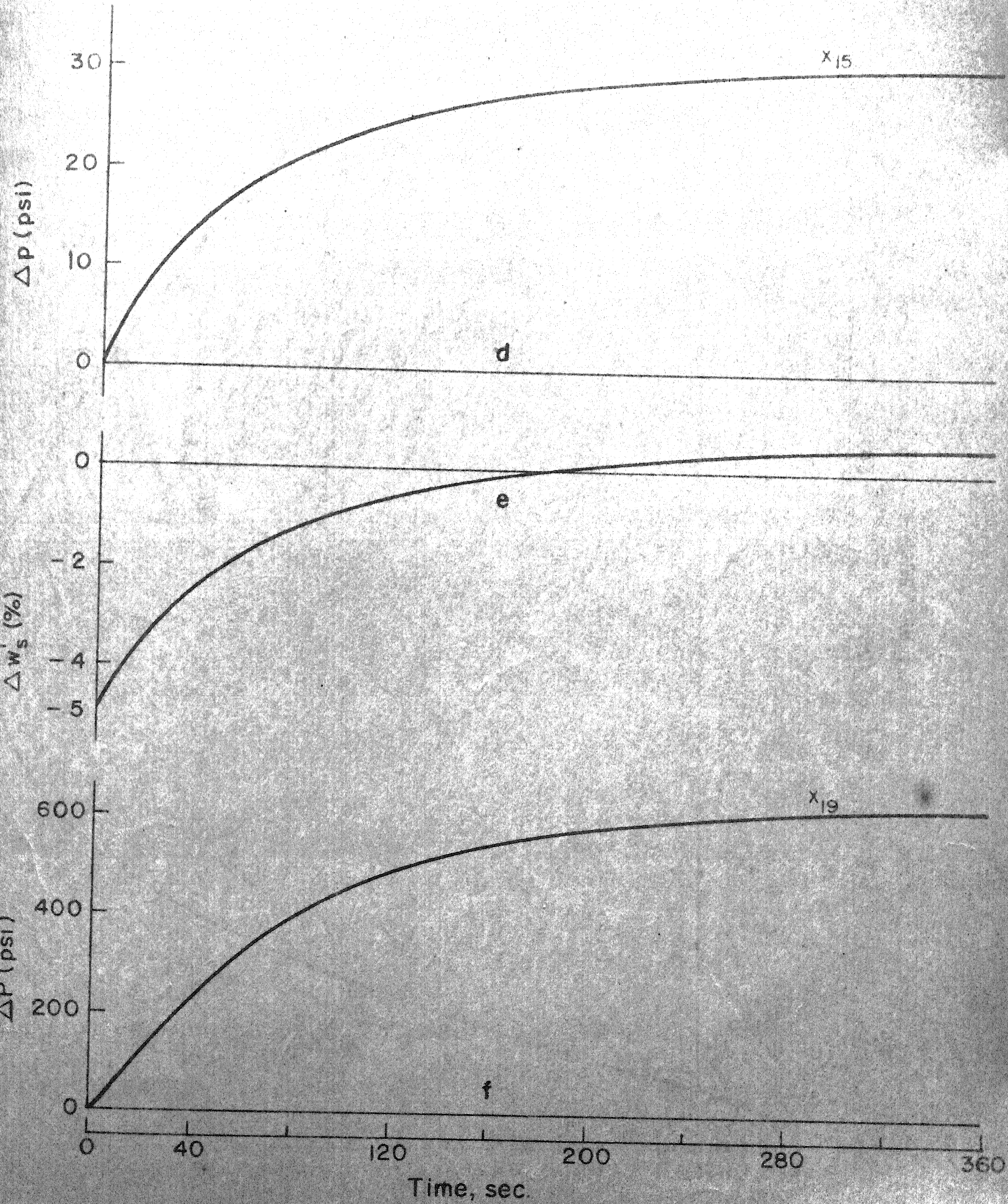
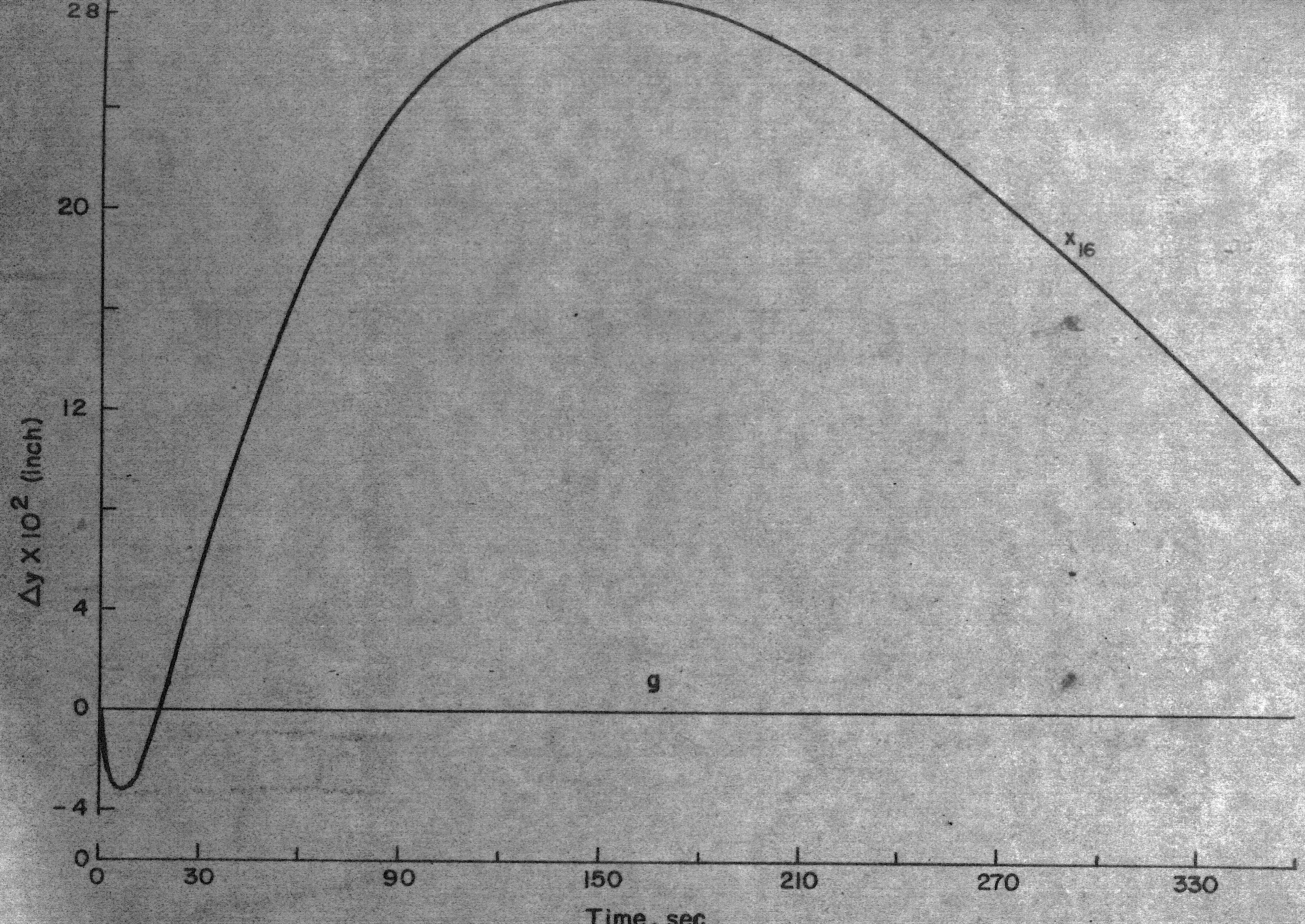


Fig. 4.1 Uncontrolled Response Of Linear Model For a −5% Step In Steam Valve Area



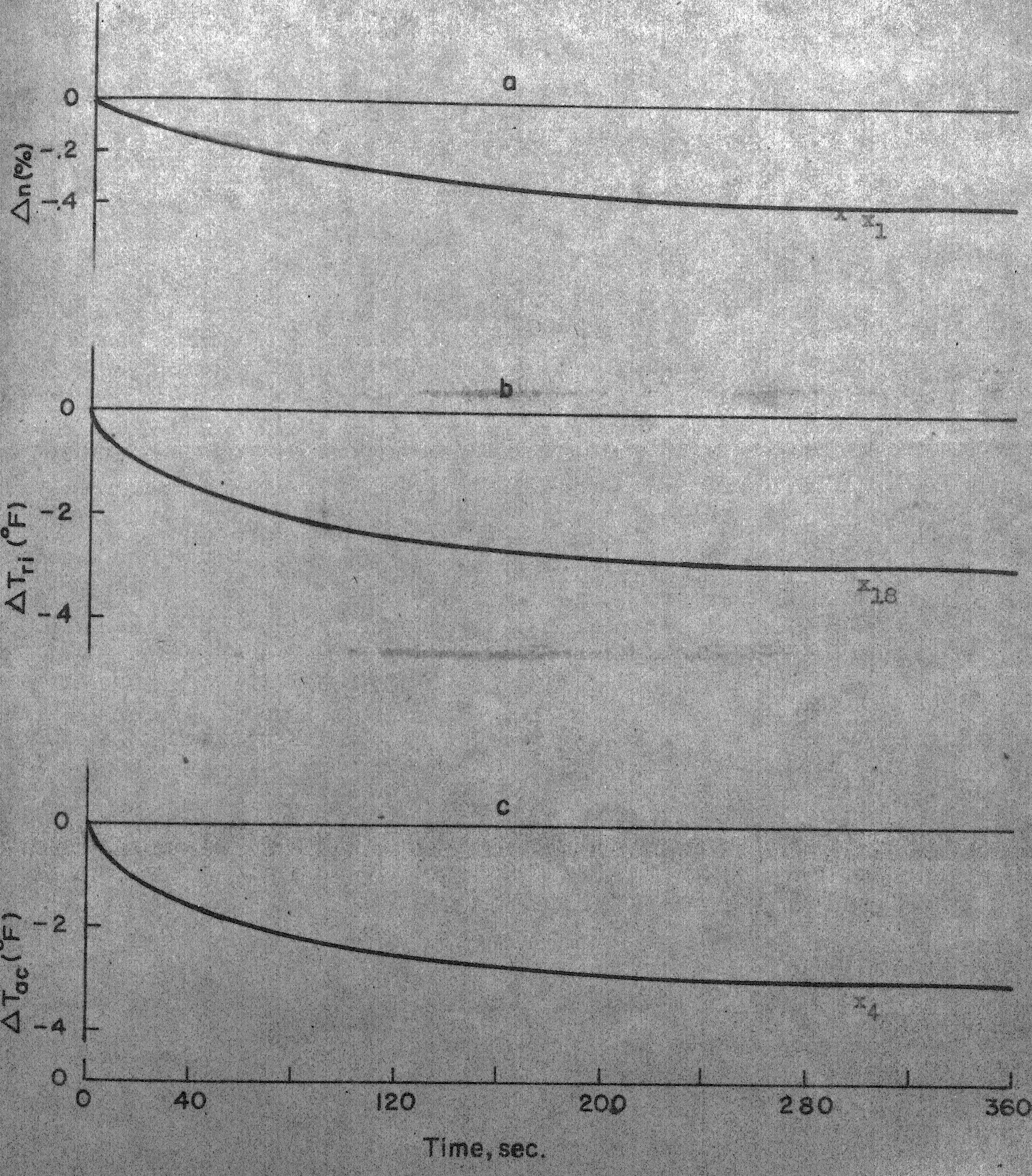
x15
30
20
10
0
Δp (psi)
d
0
-2
-4
-5
Δw's (%)
e
x19
600
400
200
0
ΔP (psi)
f
0
40
120
200
280
360
Time, sec.

Fig. 4.1 (Contd.)



28
20
12
4
0
-4
0
$\Delta y \times 10^2$ (inch)
0
30
90
150
210
270
330
Time, sec.
x_{16}
g

Fig. 4.1 (Contd.)



g.4.2 Uncontrolled Response Of Linear Model For a −15°F Step In Feed Water Inlet Temperature

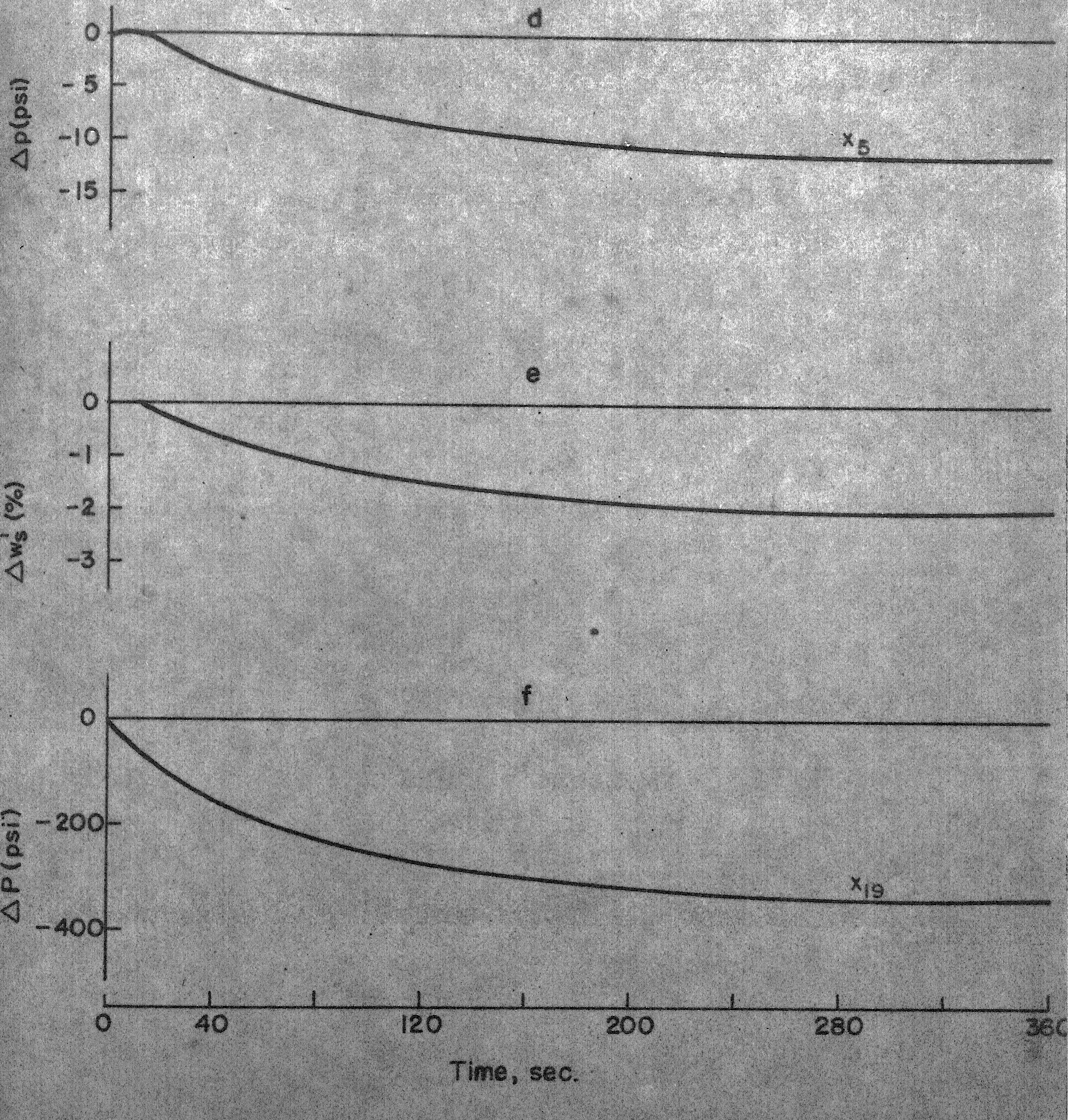

d
0
-5
-10
-15
Δp(psi)
x_5
e
0
-1
-2
-3
$\Delta w_s'$ (%)
f
0
-200
-400
ΔP (psi)
x_{19}
0
40
120
200
280
360
Time, sec.

Fig. 4.2 (Contd.)

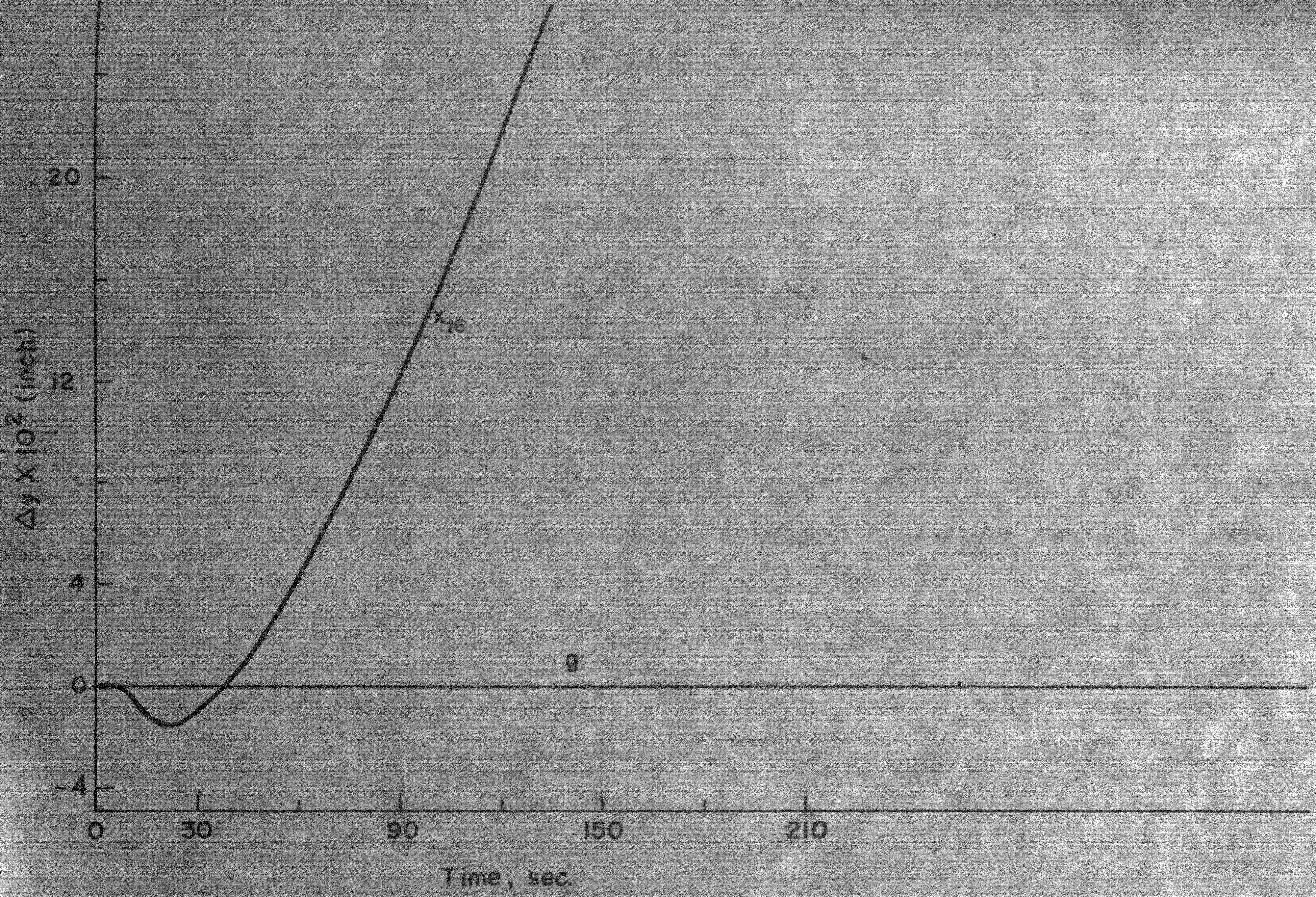


Fig. 4.2 (Contd.)

CHAPTER 5

OPTIMAL CONTROL

5.1 INTRODUCTION

The previous chapter dealt with the uncontrolled behaviour of the plant when it is acted upon by disturbances. It can be seen that the effect of the disturbance was to vary most of the system variables from their initial values. The basic requirement of the power station is to supply electrical energy as demanded by the power distribution network; this necessitates the control of a few important plant variables like the temperature of the coolant in the reactor, the pressure of the primary coolant circuit and the water level in the steam drum at prescribed values.

The control of the variables in the power station is presently accomplished by analog devices[12,6] which are usually designed to respond to an error in a single variable. A number of independent controllers are used to regulate variables such as water level in the boiler, steam pressure etc. The design of such standard controllers neglects the effects of dynamic interaction existing between the controlled variables; that is, if an error occurs in one of these variables, its controller moves a single actuator. Unfortunately, movement of this actuator causes errors in other controlled variables due to interaction among them. The new steady state operating

point is reached after a number of controllers have acted in an independent manner.

An alternative approach to plant control is to design a single multivariable controller; input to this controller consists of the desired and the actual values of the variables. If an error occurs in a plant variable, the controller responds by moving a number of actuators in a co-ordinated manner so as to reduce the error without causing unnecessary errors in other variables and thereby enabling the system to reach quickly the new steady state. The application of digital computer in power station operation for data logging, for alarm scanning and for normal start-up and shut-down introduces the possibility of developing and implementing a multivariable controller for the power station. In this chapter, optimal control theory is applied to the linear model of the pressurised heavy water reactor to design a multivariable controller.

5.2 BRIEF REVIEW OF OPTIMAL CONTROL THEORY

Analysis and design of linear multivariable controller by using optimal control theory have been well discussed in the literature[38,39]. The objective of the multivariable controller is to generate a linear constant state feedback control law which will transfer the system from a given initial state to the desired final state. In so doing the control system must satisfy the requirements relating to the performance of

the system such as the desired output, desired control effort etc., and also the implementation. The requirements on the performance is represented mathematically as a performance index. Quadratic performance index in the output or state variables and the control variables which minimises the error in the output or state variables and the control effort is most frequently used. For a linear system, the quadratic performance index yields a linear control law which is easy to ~~implement~~ compute.

The performance criterion is hence taken as

$$J = \frac{1}{2} \underline{x}^T(t_f)\underline{S}\, \underline{x}(t_f) + \frac{1}{2} \int_{t_o}^{t_f} [\underline{x}^T(t)\underline{Q}\, \underline{x}(t) + \underline{u}^T(t)\underline{R}\underline{u}(t)]\, dt \tag{5.1}$$

where $\underline{x}$ and $\underline{u}$ refer to state and control vectors,

$\underline{Q}$ is a positive semi-definite matrix,

$\underline{R}$ is a positive definite matrix,

t_o is the initial time,

and t_f is the final time.

A unique set of weighting matrices, i.e., the matrices $\underline{Q}$, $\underline{R}$ and $\underline{S}$ to satisfy the prescribed or required design specifications generally does not exist. However, the lack of uniqueness of the weighting factors does introduce a flexibility which makes selection of the performance index simpler and meaningful. Depending upon the relative importance

of the state or output variables and the control variables, the cost matrices $\underline{Q}$ and $\underline{R}$ are chosen to reflect the desired closed loop performance of the system. The constraints on the control and output variables can be indirectly taken into account by assigning suitable penalties to the constrained variables, and $\underline{Q}$ and $\underline{R}$ matrices can be chosen to reflect these penalties. For example, a large value for ith diagonal element of $\underline{R}$ means that the corresponding control variable, i.e. ith control variable, is not allowed to vary significantly.

Having selected the performance index, the design of multivariable controller can be stated as: To find the control $\underline{u}$ which minimises the performance index subject to the constraints

$$\dot{\underline{x}} = \underline{A}\,\underline{x} + \underline{B}\,\underline{u}$$

and $\quad \underline{x}\,(t_o) = \underline{x}_o$

where $\underline{x}$ is a (nxl) vector of state variables, $\underline{u}$ is a (mxl) vector of control variables, and $\underline{A}$ and $\underline{B}$ are real, constant matrices of appropriate sizes. The control which satisfies the above requirements can be obtained by the application of Pontryagin maximum principle or the Hamilton - Jacobi equation[38] and is well known in optimal control theory. The existence of the optimal control however, requires the prior investigation of the system for controllability[39] as discussed in Appendix D. The optimal control can then be obtained (as shown in Appendix E) as

$$\underline{u}(t) = -\underline{R}^{-1}\ \underline{B}^T\ \underline{P}(t)\underline{x}(t) \qquad (5.2)$$

where $\underline{P}(t)$ is the positive definite solution of

$$\dot{\underline{P}}\ (t) = -\underline{P}(t)\ \underline{A} - \underline{A}^T\underline{P}(t) + \underline{P}(t)\underline{B}\ \underline{R}^{-1}\ \underline{B}^T\ \underline{P}(t) - \underline{Q} \qquad (5.3)$$

with a terminal condition given by $\underline{P}(t_f) = \underline{S}$.
The above equation, i.e., Eq.(5.3) is known as the matrix Riccati equation and the solution of this equation provides the matrix $\underline{P}$ to calculate the feedback control law through Eq .(5.2). This problem of finding optimal control policy for a finite t_f is known as the finite time linear regulator problem.

When t_f in Eq. (5.1) is infinite, $\underline{P}$ can be obtained by solving the algebraic matrix Riccati equation, i.e. the solution of

$$\underline{P}\ \underline{A} + \underline{A}^T\underline{P} - \underline{P}\ \underline{B}\ \underline{R}^{-1}\ \underline{B}^T\ \underline{P} + \underline{Q} = \underline{0}$$

gives the matrix $\underline{P}$ to calculate the control law. Also $\underline{P}$ is independent of time. This problem is known as infinite time linear regulator problem. The procedures for solving the algebraic and the differential matrix Riccati equations are discussed in the following sections.

5.2.1 Infinite Time Regulator Problem

Infinite time regulator problem requires the solution of the algebraic matrix Riccati equation which is obtained by the method of successive approximation[40,41]. In this

method, an initial choice of $\underline{P}$ is made such that the controlled system given by the equation

$$\dot{\underline{x}} = (\underline{A} - \underline{B}\ \underline{R}^{-1}\ \underline{B}^T\ \underline{P})\ \underline{x} \tag{5.4}$$

is stable. This initial value of $\underline{P}$ may be guessed or may be obtained by integrating Eq. (5.3) backwards in time till a $\underline{P}$ which makes the controlled system stable is obtained. Then an iterative scheme is followed to obtain $\underline{P}$ at the ith iteration from its value at the (i-1)th iteration. At the ith iteration, $\underline{P}$ is obtained as the solution of

$$\underline{P}^i\ \underline{A}^i + (\underline{A}^i)^T\ \underline{P}^i + \underline{Q}^i = \underline{0}$$

where $\underline{A}^i = \underline{A} - \underline{B}\ \underline{R}^{-1}\ \underline{B}^T\ \underline{P}^{i-1}$

and $\underline{Q}^i = \underline{Q} + \underline{P}^{i-1}\ \underline{B}\ \underline{R}^{-1}\ \underline{B}^T\ \underline{P}^{i-1}$

These iterations are carried out until the difference between the elements of successive $\underline{P}$ matrices is within the required accuracy. In this work, the iterations are continued till the elements of P matrices at successive iterations agreed upto 1 part in 10^5.

5.2.2 Finite Time Regulator Problem

This requires the solution of the matrix Riccati equation in its differential form, i.e., Eq. (5.3). The solution is obtained by direct integration of the equation backward in time, starting from the final time t_f with $\underline{P}(t_f)=\underline{S}$ till the initial time t_o is reached. The matrices $\underline{P}(t)$ thus

obtained at various time intervals are stored and are used for generating the feedback control given by Eq.(5.2). It can be seen that the finite time regulator problem results in a time varying control law; the implementation of the feedback law then requires a programmed control and becomes difficult. Therefore in this thesis, this is not attempted and the infinite time regulator theory is used to obtain a time invariant control law for the pressurised heavy water reactor.

To summarise, the problem of calculating the feedback control law for the pressurised water reactor can be treated as a linear regulator problem. The infinite time regulator problem which yields a time invariant control law, can be stated as; Given

$$\dot{\underline{x}} = \underline{A}\,\underline{x} + \underline{B}\,\underline{u} \quad , \quad \underline{x}(0) = \underline{x}_0$$

it is required to find the control $\underline{u}$ which minimises the performance index

$$J = \tfrac{1}{2}\int_0^\infty [\underline{x}^T(t)\underline{Q}\,\underline{x}(t) + \underline{u}^T(t)\,\underline{R}\,\underline{u}(t)]\,dt$$

where $\underline{Q}$ and $\underline{R}$ are appropriate matrices. The solution to the above problem is written as

$$\underline{u}(t) = \underline{K}\,\underline{x}(t)$$

$$= -\underline{R}^{-1}\,\underline{B}^T\,\underline{P}\,\underline{x}(t)$$

where $\underline{K}$ is the state feedback matrix and is equal to $-\underline{R}^{-1}\underline{B}^T\underline{P}$ and where $\underline{P}$ is the positive definite symmetric matrix satisfying the following matrix Riccati equation

$$\underline{P}\,\underline{A} + \underline{A}^T\,\underline{P} - \underline{P}\,\underline{B}\,\underline{R}^{-1}\,\underline{B}^T\,\underline{P} + \underline{Q} = \underline{0}$$

The state feedback matrix $\underline{K}$ is then used to generate the control vector $\underline{u}(t)$ from the state vector $\underline{x}(t)$.

5.3 CONTROL STUDIES ON THE PRESSURISED HEAVY WATER REACTOR

The infinite time regulator theory described in the preceeding section is applied to develop proportional controller and proportional-integral controller for the pressurised heavy water reactor. The following sections describe the performance of the reactor with these controllers.

5.3.1 Proportional Controller

The dynamics of the linear model of the pressurised heavy water reactor has been represented by Eq.(4.7) which is repeated below for convenience,

$$\dot{\underline{x}} = \underline{A}\,\underline{x} + \underline{B}_1\,\underline{u} + \underline{B}_2\,\underline{w} \tag{4.7}$$

The design and operational requirements of the reactor impose minimum variations of the following three state variables: i) average temperature of coolant x_4 ii) level of water in the steam drum x_{16} and iii) pressure of the primary heat transport system x_{19}. In order to meet these specifications, the weighting matrices $\underline{Q}$ and $\underline{R}$ are chosen as

$$\underline{Q} = \text{diag}\,(0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0.1, 0, 0, 0.1) \tag{5.5}$$

$$\underline{R} = \mathrm{diag}\ (1,\ 1,\ 1) \tag{5.6}$$

where the performance index J is given by

$$J = \frac{1}{2} \int_0^\infty [\underline{x}^T(t)\ \underline{Q}\ \underline{x}(t) + \underline{u}^T(t)\ \underline{R}\ \underline{u}(t)]\ dt$$

The minimisation of J using infinite time regulator theory leads to

$$\underline{u}(t) = \underline{K}\ \underline{x}(t) \tag{5.7}$$

where the elements of the feedback matrix $\underline{K}$ are given in Table 5.1.

Table 5.1

Matrix $\underline{K}$: (3x19 matrix)

K(1,1) = -0.9973E-01	K(2,1) = -0.1388E-02	K(3,1) = -0.8248E-02
K(1,2) = -0.9234E-00	K(2,2) = -0.1713E-01	K(3,2) = -0.7908E-01
K(1,3) = -0.8569E-00	K(2,3) = -0.9041E-02	K(3,3) = -0.7466E-01
K(1,4) = +0.1164E+02	K(2,4) = -0.4163E+01	K(3,4) = +0.1900E+01
K(1,5) = +0.1237E+01	K(2,5) = -0.1530E+01	K(3,5) = +0.1196E+00
K(1,6) = -0.3628E+00	K(2,6) = -0.2839E+00	K(3,6) = -0.2001E+00
K(1,7) = +0.2222E+01	K(2,7) = +0.7054E+01	K(3,7) = -0.5945E+00
K(1,8) = -0.3200E+00	K(2,8) = -0.1764E+01	K(3,8) = +0.2280E+00
K(1,9) = -0.1356E+01	K(2,9) = -0.9120E+00	K(3,9) = -0.2917E-01
K(1,10)= +0.2575E+00	K(2,10)= +0.1432E+01	K(3,10)= -0.1258E+00
K(1,11)= +0.5238E+01	K(2,11)= -0.1852E+01	K(3,11)= +0.8560E+00
K(1,12)= -0.1686E+00	K(2,12)= -0.1827E+00	K(3,12)= +0.9244E-02
K(1,13)= -0.1719E+00	K(2,13)= -0.4355E+00	K(3,13)= +0.5022E-01
K(1,14)= +0.5599E-01	K(2,14)= +0.2657E+00	K(3,14)= +0.3455E-02
K(1,15)= -0.8207E+00	K(2,15)= +0.4305E+00	K(3,15)= -0.1099E+00
K(1,16)= -0.9063E-01	K(2,16)= -0.3082E+00	K(3,16)= +0.2381E-01
K(1,17)= -0.1287E+01	K(2,17)= -0.3317E+01	K(3,17)= +0.4150E+00
K(1,18)= -0.1142E+02	K(2,18)= +0.1027E+01	K(3,18)= -0.1220E+01
K(1,19)= -0.3013E+00	K(2,19)= +0.8652E-01	K(3,19)= -0.4956E-01

[The notation used to represent the elements of $\underline{K}$ is the same as that used in Chapter 4].

Performance of the system with this feedback control is obtained for a constant disturbance of -5 percent change in the area of steam valve w_1. Implementation of this control is seen to produce stable closed loop responses (Fig.5.1) but the system settles with undesirable steady state off-sets. These off-sets depend significantly on the weighting factors used. Further the off-sets are to be expected since the performance index does not take into account the physically necessary off-sets which must exist in the variables following a sustained change in the disturbance vector[10]. However the steady state off-sets can be eliminated by the method discussed in the following section.

5.3.2 Proportional Controller Based on Error Coordinates

The steady state off-sets obtained in the previous section can be eliminated by modifying the performance index. A new performance index is formulated[10] as

$$J = \frac{1}{2} \int_0^\infty [(\underline{x}(t)-\underline{x}_s)^T \underline{Q} (\underline{x}(t)-\underline{x}_s) + (\underline{u}(t)-\underline{u}_s)^T \underline{R} (\underline{u}(t) - \underline{u}_s)] dt \qquad (5.8)$$

In Eq.(5.8) $\underline{x}_s$ and $\underline{u}_s$ are the equilibrium values after a step change in the disturbance vector.

5.3.2.1 Calculation of Steady State Values

The steady state values $\underline{x}_s$, $\underline{u}_s$ are determined from Eq.(4.7). i.e., the steady state of the system after a step

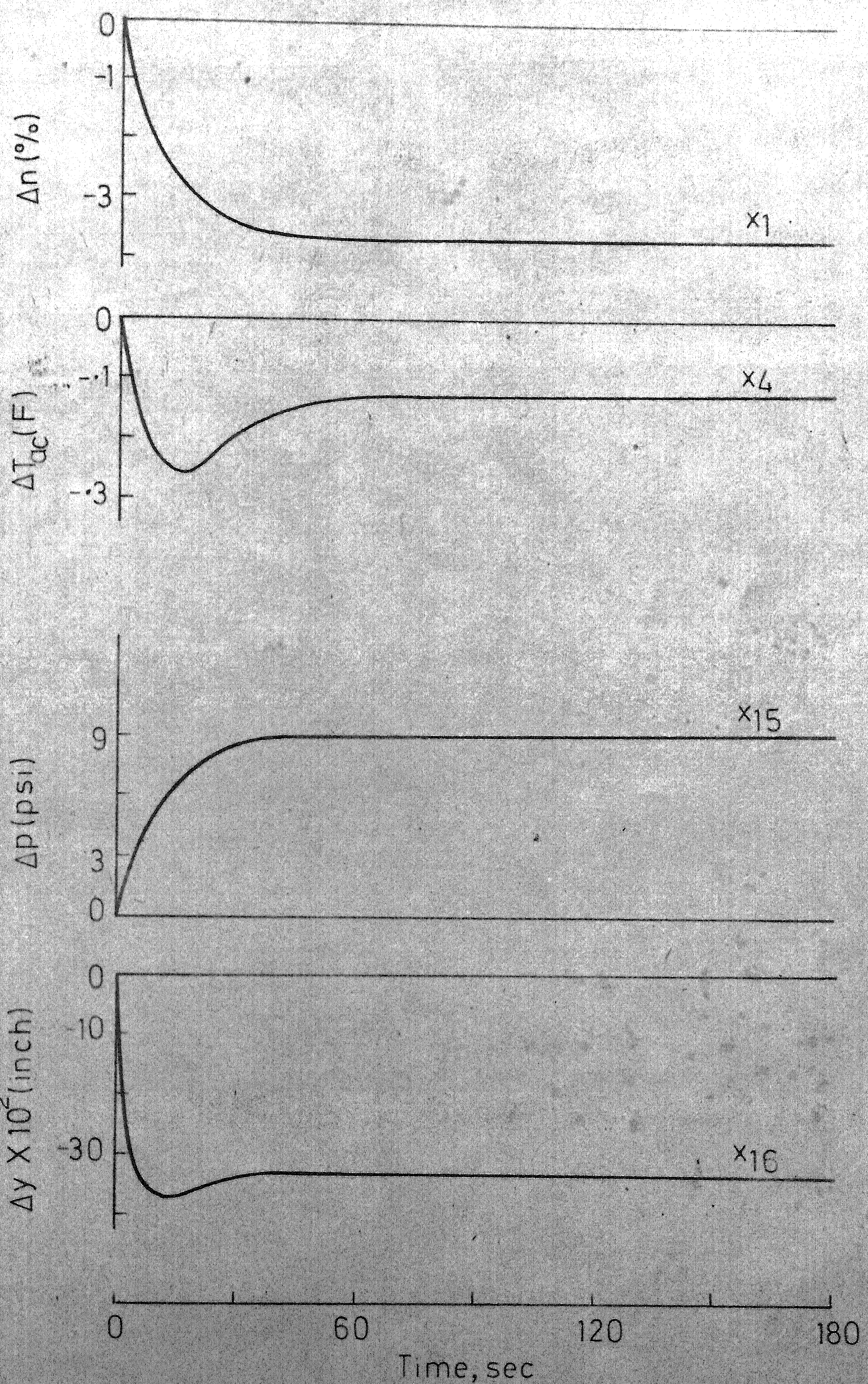


FIG. 5.1 RESPONSE OF LINEAR MODEL WITH PROPORTIONAL STATE FEEDBACK

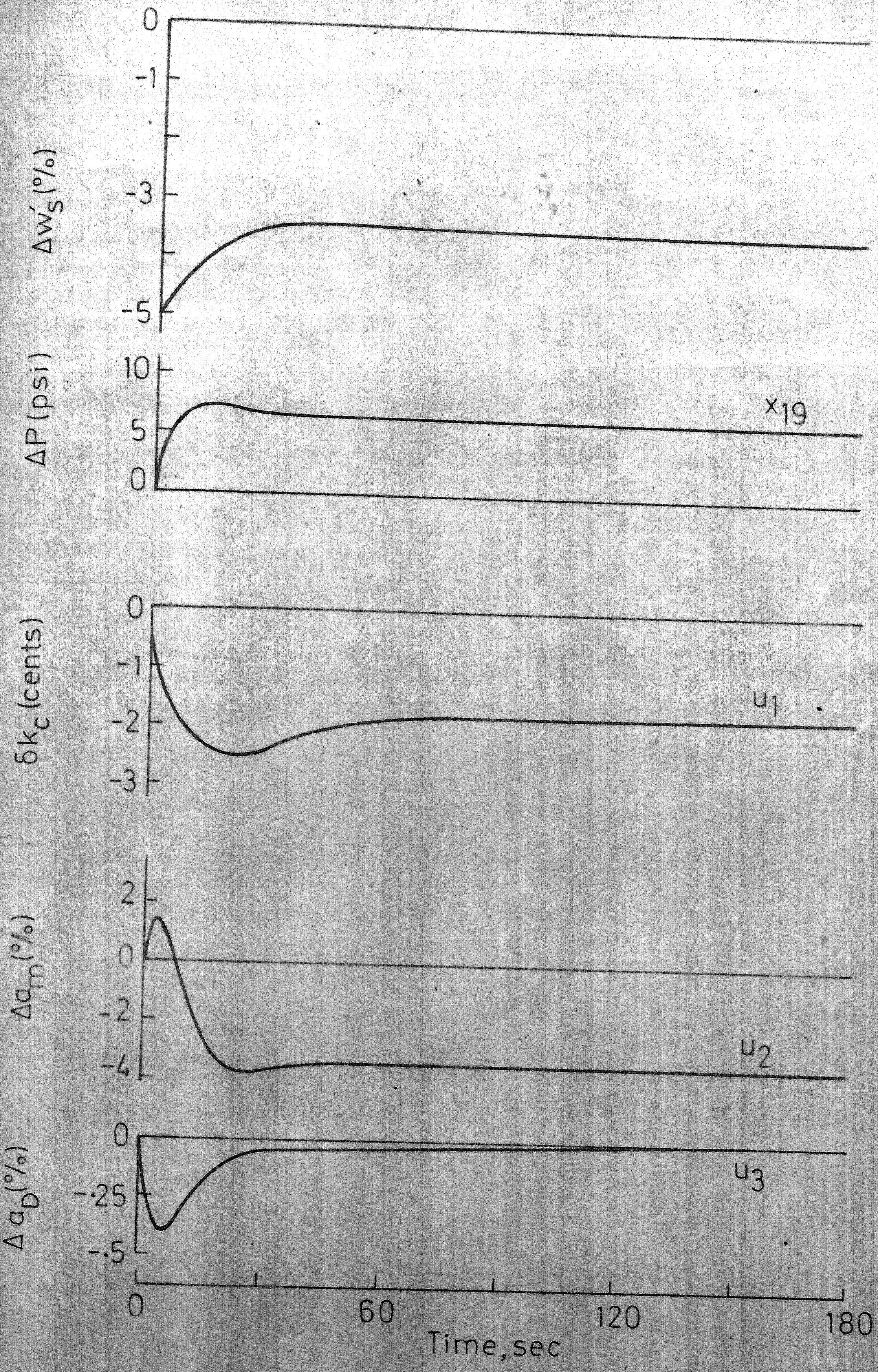


FIG. 5·1 (CONTINUED)

change in $\underline{w}$ can be obtained from Eq.(4.7) as the solution of

$$\dot{\underline{x}}_s = \underline{A}\ \underline{x}_s + \underline{B}_1\ \underline{u}_s + B_2\ \underline{w} = \underline{0} \qquad (5.9)$$

Having specified $\underline{w}$, it can be seen that the above equation has more unknowns than the number of equations. This allows for certain unknowns to be specified arbitrarily as required by the needs and design considerations. In this pressurised heavy water reactor study, there are 22 variables (19 state and 3 control variables) in 19 equations. Hence three variables, either state or control, can be specified. Furthermore, since there are three control variables, three of the states can be driven to zero, i.e. to their initial values. These three states are chosen as the average temperature of coolant in reactor x_4, the level of water in steam drum x_{16} and the pressure of the primary heat transport system x_{19} and it is required to hold their magnitudes at their respective equilibrium levels prior to the application of the disturbance. Thus the steady state values of x_4, x_{16} and x_{19} are specified as zero and Eq.(5.9) is solved for other unknowns. The solution then provides the necessary steady state values of the control variables so as to maintain x_4, x_{16} and x_{19} at their prescribed values in the presence of a disturbance.

For a -5 percent step in the area of steam valve w_1, the above procedure yields the following values for $\underline{x}_s$ and $\underline{u}_s$

$$\underline{x}_s^T = -(3.5196,\ 3.5196,\ 19.5617,\ 0.0,\ -1.0310,\ -1.5318,\ 3.0295,\ -1.4617,\ -1.6043,\ -5.6431,\ -1.4076,\ -1.1110,\ -6.3133,\ -1.0135,\ -9.5081,\ 0.0,\ 1.4076,\ -1.4076,\ 0.0)$$

and $\underline{u}_s^T = -(1.6815,\ 3.3011,\ 0.0)$

5.3.2.2 Dynamics Based on Error Variables

The dynamics of the reactor is now based on the errors between the steady state values $\underline{x}_s$ and $\underline{u}_s$ and the instantaneous values of $\underline{x}(t)$ and $\underline{u}(t)$. Equations (4.7) and (5.9) are combined to yield

$$\dot{\underline{x}}(t) - \dot{\underline{x}}_s = \underline{A}\,[\underline{x}(t) - \underline{x}_s] + \underline{B}_1\,[\underline{u}(t) - \underline{u}_s]$$

Define vectors $\underline{e}$ and $\underline{f}$ as

$$\underline{e}(t) = \underline{x}(t) - \underline{x}_s$$

and $$\underline{f}(t) = \underline{u}(t) - \underline{u}_s$$

Then the above equation can be written as

$$\dot{\underline{e}}(t) = \underline{A}\,\underline{e}(t) + \underline{B}_1\,\underline{f}(t)$$

and the performance index Eq.(5.8) is written as

$$J = \frac{1}{2}\int_0^\infty [\underline{e}^T(t)\,\underline{Q}\,\underline{e}(t) + \underline{f}^T(t)\,\underline{R}\,\underline{f}(t)]\,dt.$$

The application of linear regulator theory to the above system of equations with the same $\underline{Q}$ and $\underline{R}$ matrices as given by Eqs. (5.5) and (5.6) leads to the following control law

$$\underline{f} = \underline{K}\,\underline{e}$$

The elements of the feedback matrix $\underline{K}$ are the same as that of Eq.(5.7) which are given in Table 5.1. Transforming the control law

$$\underline{f}(t) = \underline{K}\ \underline{e}(t)$$

to the original co-ordinates $\underline{x}$ and $\underline{u}$, the control $\underline{u}$ is written as

$$\underline{u}(t) = \underline{u}_s + \underline{K}\ (\underline{x}(t)-\underline{x}_s) \qquad (5.10)$$

The performance of the system with the above control law for a -5 percent step in w_1 is shown in Figure 5.2. It can be seen that this control law has produced closed loop responses with complete elimination of the steady state off-sets in the selected variables x_4, x_{16} and x_{19}. The response of x_4, i.e. the average temperature of the coolant in the reactor, is obtained for a large period of time and is compared with the response obtained with the control law given by Eq.(5.7) (Figure 5.3). Comparison of figures 5.1 and 5.2 show that the initial values for the controls, Δa_m and Δa_D are quite different for the two control schemes, i.e. Eqs.(5.7) and (5.10), with the control law given by Eq.(5.10) requiring large initial values for Δa_m and Δa_D.

5.3.2.3 Drawbacks of the Controller

The control law represented by Eq.(5.10) is theoretically optimal for step disturbance. However, it is quite difficult to implement because of the following two drawbacks.

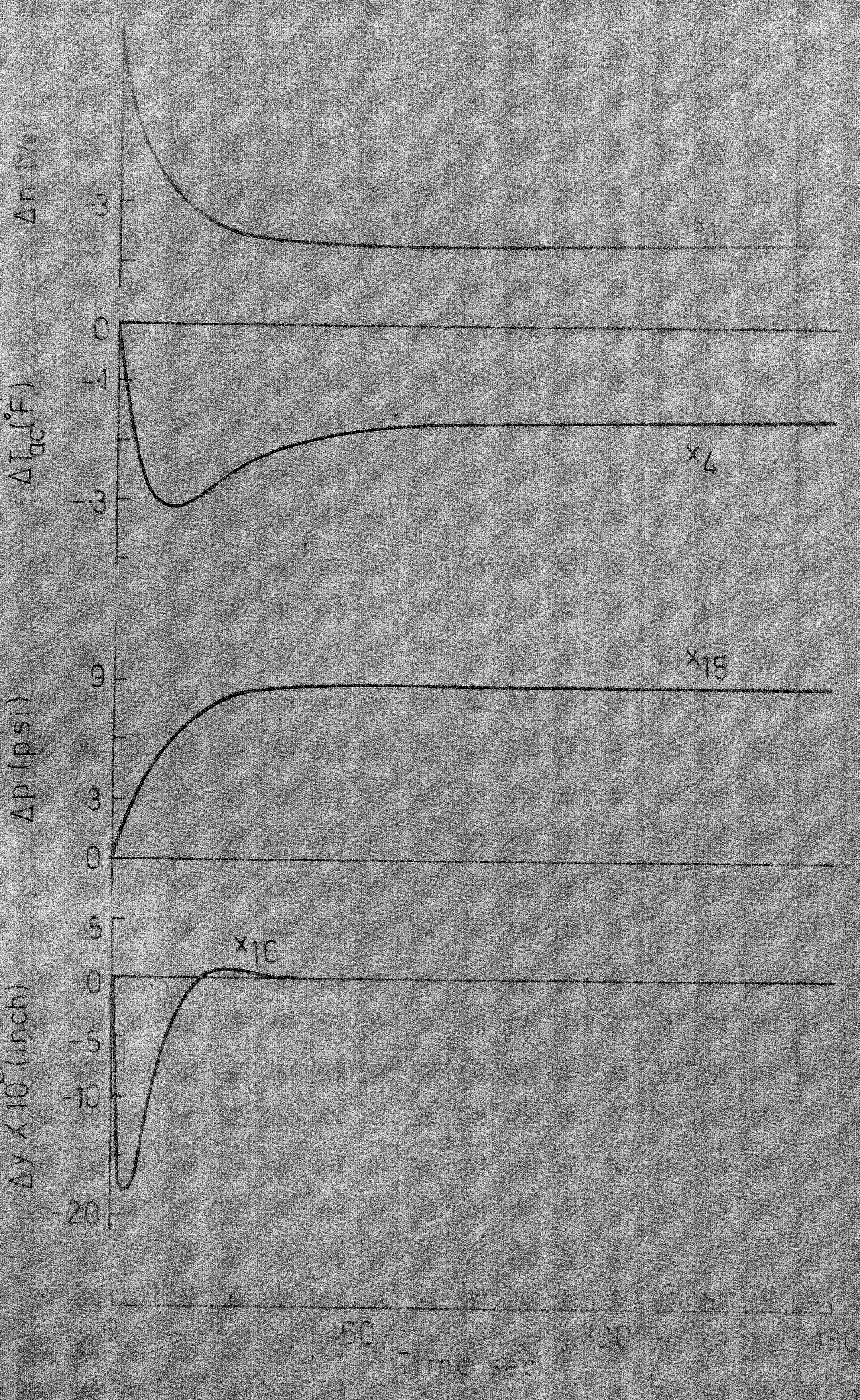


FIG. 5.2 RESPONSE OF LINEAR MODEL WITH PROPORTIONAL ERROR CO-ORDINATE STATE FEEDBACK

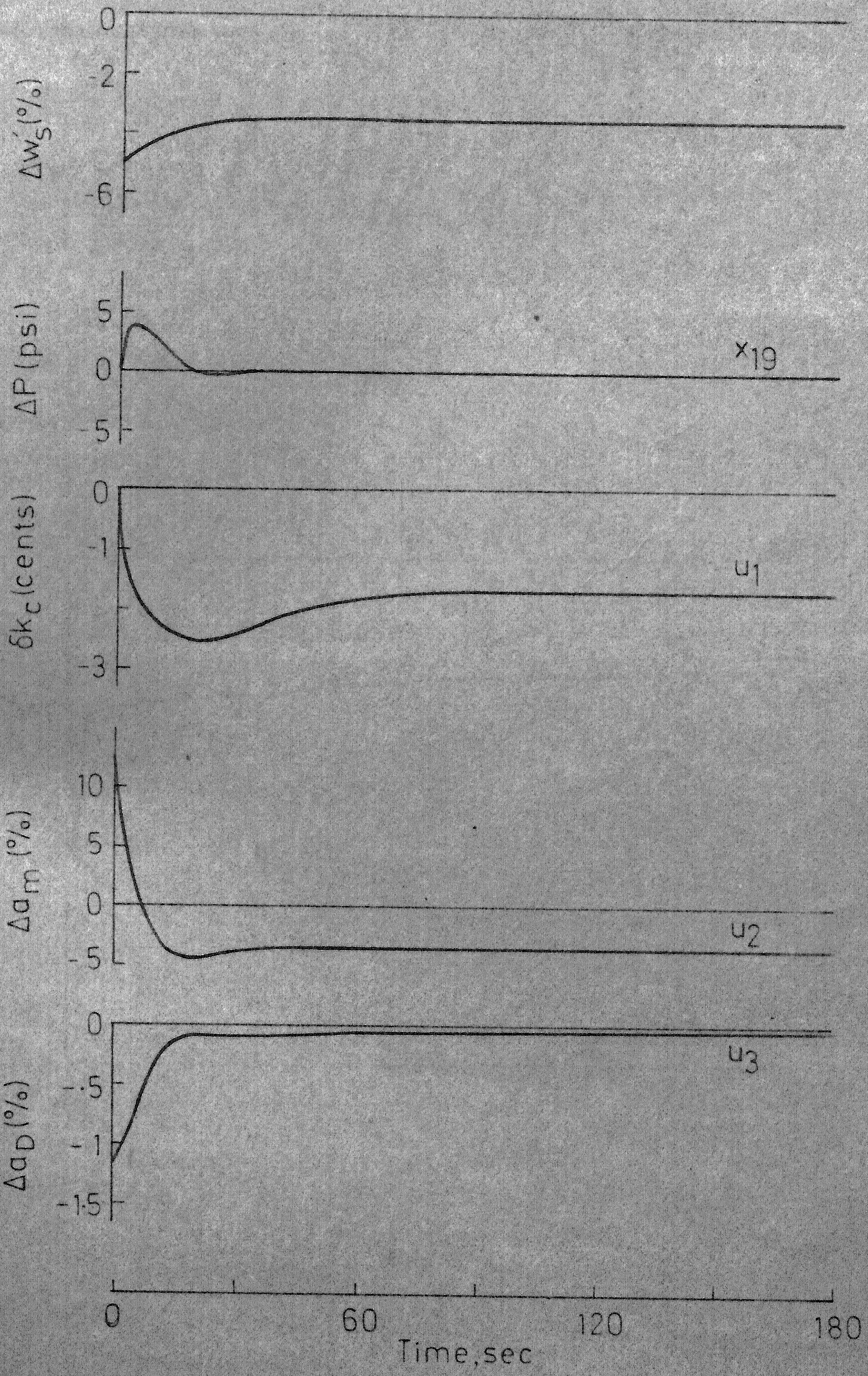


FIG. 5.2 (CONTINUED)

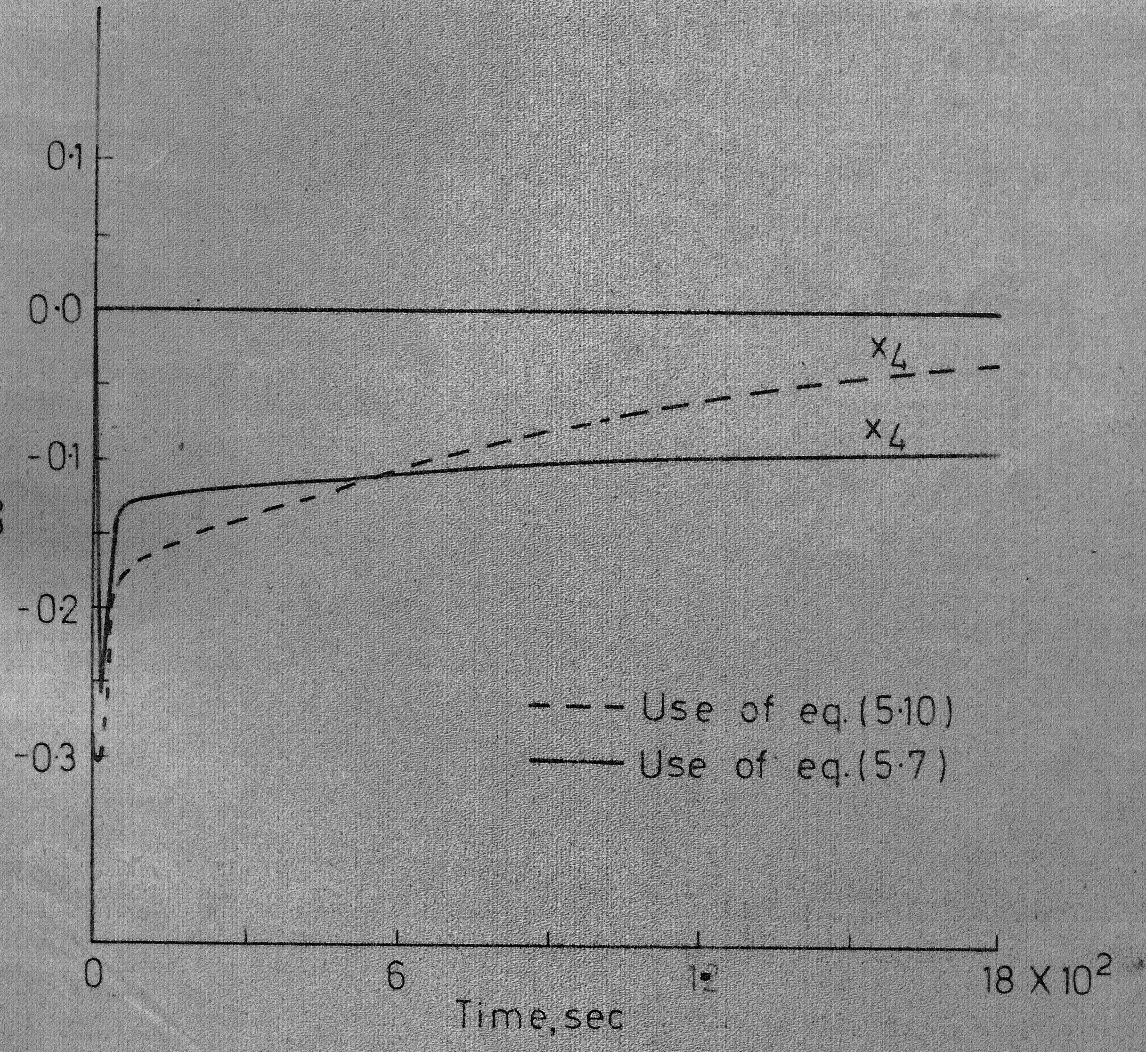


FIG. 5·3 RESPONSE OF AVERAGE TEMPERATURE OF COOLANT IN THE REACTOR

(a) Equation (5.10) needs a knowledge of the nature of the step disturbance and its magnitude so that the steady state vectors $\underline{x}_s$ and $\underline{u}_s$ can be evaluated.

(b) This control law is sensitive to the parameters of the system. This is mainly due to the use of the terms $(\underline{x} - \underline{x}_s)$ and $(\underline{u} - \underline{u}_s)$ in the performance index which forces $\underline{u}$ and $\underline{x}$ to approach the calculated steady state values $\underline{u}_s$ and $\underline{x}_s$. Since $\underline{x}_s$ and $\underline{u}_s$ are obtained from Eq.(4.7), their magnitudes depend on the matrices $\underline{A}$, $\underline{B}_1$ and $\underline{B}_2$. If the plant parameters differ from the nominal values, i.e., the matrices $\underline{A}$, $\underline{B}_1$ and $\underline{B}_2$ are different, then the control law given by Eq.(5.10) will drive the system to a steady state where the requirement of zero steady state off-sets in the selected variables may not be fulfilled.

The shortcomings of the above control scheme are mainly due to the use of proportional controller in the presence of a constant disturbance. It can be visualised that a proportional-integral controller may be able to meet the requirements of zero steady state off-sets in selected variables when the system is subjected to constant disturbances. Since linear regulator theory leads to proportional controller using state variable feedback, modifications of the regulator theory is needed to yield a proportional integral controller[42]. The

following section discusses the design of the proportional - integral controller using linear regulator theory.

5.3.3 Proportional-Integral Controller

In section 5.3.2, optimal control laws are calculated based on performance index which penalise the deviation of the state and control vectors from their steady state values. It is also seen that the steady state values depend on the nominal values of the plant parameters. If the plant parameters are different from the nominal values, then $\underline{x}$ and $\underline{u}$ would not converge to their calculated steady state values. This is a disadvantage with the proportional controller of section 5.3.2. However this drawback may be eliminated by noting that $\underline{x}$ and $\underline{u}$ do tend to some constant values so that their time derivatives tend to zero regardless of the actual values. The way that this can be incorporated into the optimal regulator theory has been discussed by M. Athans[42], Parker[43] and Smith and Davison[44] and the procedure is now described.

5.3.3.1 Modified Linear Regulator Theory[43,44]

Consider the system represented by

$$\dot{\underline{x}} = \underline{A}\,\underline{x} + \underline{B}_1\,\underline{u} + B_2\,\underline{w} \qquad (5.11)$$

$$\underline{y} = \underline{C}\underline{x} + \underline{D}_1\,\underline{u} + D_2\,\underline{w} \qquad (5.12)$$

with $\underline{x}(o) = \underline{x}_o$, and $\underline{u}(o) = \underline{u}_o$ (5.13)

where $\underline{x}$ is a (nxl) vector of state variables

$\underline{u}$ is a (mxl) vector of control variables

and $\underline{w}$ is a (qxl) vector of **disturbance variables**

Further $\underline{y}$ is a (mxl) vector of output variables whose values are to be held zero. $\underline{A}$, $\underline{B}_1$, B_2, $\underline{C}$, $\underline{D}_1$ and $\underline{D}_2$ are real constant matrices of appropriate dimensions.

Define vectors $\underline{z}$ and $\underline{v}$ as

$$\underline{z}^T = (\dot{\underline{x}}^T, \underline{y}^T) \tag{5.14}$$

$$\underline{v} = \dot{\underline{u}} \tag{5.15}$$

By differentiation, Eqs.(5.11) and (5.12) are transformed as

$$\dot{\underline{z}} = \tilde{\underline{A}}\, \underline{z} + \tilde{\underline{B}}\, \underline{v} \tag{5.16}$$

$$\underline{z}(0) = \begin{bmatrix} \underline{A} & \underline{B}_1 & \underline{B}_2 \\ \underline{C} & \underline{D}_1 & \underline{D}_2 \end{bmatrix} \begin{bmatrix} \underline{x}_0 \\ \underline{u}_0 \\ \underline{w} \end{bmatrix} \tag{5.17}$$

where $\tilde{\underline{A}} = \begin{bmatrix} \underline{A} & \underline{0} \\ \underline{C} & \underline{0} \end{bmatrix}$

and $\tilde{\underline{B}} = \begin{bmatrix} \underline{B}_1 \\ \underline{D}_1 \end{bmatrix}$

With the performance index defined as

$$J = \tfrac{1}{2} \int_0^\infty [\underline{z}^T\, \underline{Q}\, \underline{z} + \underline{v}^T\, \underline{R}\, \underline{v}]\ dt \tag{5.18}$$

where Q and R matrices are of appropriate dimensions, a stable linear state feedback control law may now be obtained for the transformed system represented by Eq.(5.16). The control is then of the form

$$\underline{v} = \underline{K}_1 \underline{z} \tag{5.19}$$

which on transformation into the original variables and by partitioning of the matrix $\underline{K}_1$ becomes

$$\dot{\underline{u}} = \underline{K}\,\dot{\underline{x}} + \underline{L}\,\underline{y} \tag{5.20}$$

On integrating the above equation, one obtains

$$\underline{u}(t) = \underline{K}\,\underline{x}(t) + \underline{L} \int_0^t \underline{y}\, ds \tag{5.21}$$

In obtaining Eq.(5.21), the initial conditions of $\underline{x}$, $\underline{u}$ and $\underline{y}$ have been taken as zero. Equation (5.21) provides a control law with proportional feedback of the state variables and integral feedback of the output variables which are to be driven to zero as time tends to infinity. The above technique is now used to obtain a proportional - integral controller for the pressurised heavy water reactor and is discussed in the following section.

5.3.3.2 Application on the Pressurised Heavy Water Reactor

The dynamics of the linear model of the reactor has been represented by Eq.(4.7) which is repeated for convenience

$$\dot{\underline{x}} = \underline{A}\,\underline{x} + \underline{B}_1\,\underline{u} + \underline{B}_2\,\underline{w} \tag{4.7}$$

Since the design and operational requirements of the reactor need that the variations in the average temperature of the coolant in the reactor, the level of water in the steam drum and the pressure of primary heat transport circuit are to be minimum, the output vector $\underline{y}$ becomes

$$\underline{y}^T = (x_4, x_{16}, x_{19}) \tag{5.22}$$

Hence for this $\underline{y}$, the matrices $\underline{C}$, $\underline{D}_1$, $\underline{D}_2$ in Eq.(5.12) are such that all the elements are zero excepting

$$c_{1,4} = c_{2,16} = c_{3,19} = 1.0 \tag{5.23}$$

The weighting matrices in the performance index given by Eq.(5.18) are chosen as

$$\underline{Q} = \text{diag}\ (0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0.1, 0.1) \tag{5.24}$$

and $$\underline{R} = \text{diag}\ (1, 1, 1) \tag{5.25}$$

The modified linear regulator theory is now applied to the system represented by the equations (4.7), (5.23),(5.24) and (5.25). The control law thus obtained, is given as

$$\underline{u}(t) = \underline{K}\ \underline{x}(t) + \underline{L} \int_0^t \underline{y}\ ds \tag{5.26}$$

where the matrices $\underline{K}$ and $\underline{L}$ are shown in Tables 5.2 and 5.3.

The above feedback control law is used to study the closed loop dynamic behaviour of the pressurised heavy water reactor for a −5 percent step change in the area of steam valve w_1. The performance of the system is quite satisfactory (Fig.5.4). The output variables x_4, x_{16} and x_{19} reach zero magnitudes within a minute of the occurrence of the disturbance.

In order to study the effect of the weight on the average temperature of the coolant in the reactor, the matrix $\underline{Q}$ given by Eq.(5.24) is modified as

Table 5.2

Matrix K: (3x19 Matrix

K(1, 1) = -0.154E+00	K(2, 1) = -0.426E-02	K(3, 1) = -0.142E-01
K(1, 2) = -0.990E+00	K(2, 2) = -0.351E-02	K(3, 2) = -0.352E-01
K(1, 3) = -0.139E+01	K(2, 3) = -0.416E-01	K(3, 3) = -0.134E+00
K(1, 4) = +0.175E+02	K(2, 4) = -0.112E+02	K(3, 4) = +0.164E+02
K(1, 5) = -0.308E+01	K(2, 5) = -0.180E+02	K(3, 5) = +0.332E+01
K(1, 6) = -0.458E+01	K(2, 6) = -0.114E+02	K(3, 6) = -0.476E-01
K(1, 7) = +0.487E+01	K(2, 7) = +0.267E+02	K(3, 7) = -0.233E+01
K(1, 8) = -0.158E+01	K(2, 8) = -0.795E+01	K(3, 8) = +0.326E+01
K(1, 9) = -0.297E+01	K(2, 9) = -0.629E+01	K(3, 9) = +0.259E+00
K(1,10) = +0.397E+00	K(2,10) = +0.342E+00	K(3,10) = -0.293E+00
K(1,11) = +0.790E+01	K(2,11) = -0.420E+01	K(3,11) = +0.761E+01
K(1,12) = -0.282E+00	K(2,12) = -0.622E+00	K(3,12) = +0.331E-01
K(1,13) = -0.235E+00	K(2,13) = -0.158E+01	K(3,13) = +0.140E+00
K(1,14) = +0.770E+00	K(2,14) = +0.274E+01	K(3,14) = -0.110E+00
K(1,15) = +0.332E+00	K(2,15) = +0.698E+01	K(3,15) = -0.857E+00
K(1,16) = -0.266E+00	K(2,16) = -0.133E+01	K(3,16) = +0.103E+00
K(1,17) = -0.421E+01	K(2,17) = -0.192E+02	K(3,17) = +0.653E+01
K(1,18) = -0.188E+02	K(2,18) = +0.492E+01	K(3,18) = -0.995E+00
K(1,19) = -0.517E+00	K(2,19) = +0.222E+00	K(3,19) = -0.353E+00

Table 5.3

Matrix L: (3x3 Matrix)

L(1, 1) = -0.165E+00	L(2, 1) = +0.794E-01	L(3, 1) = 0.981E+00
L(1, 2) = -0.811E-01	L(2, 2) = -0.305E+00	L(3, 2) =+0.110E-01
L(1, 3) = -0.301E+00	L(2, 3) = +0.778E-01	L(3, 3) =-0.570E-01

[The notation used to represent the elements of K is the same as that used in Chapter 4]

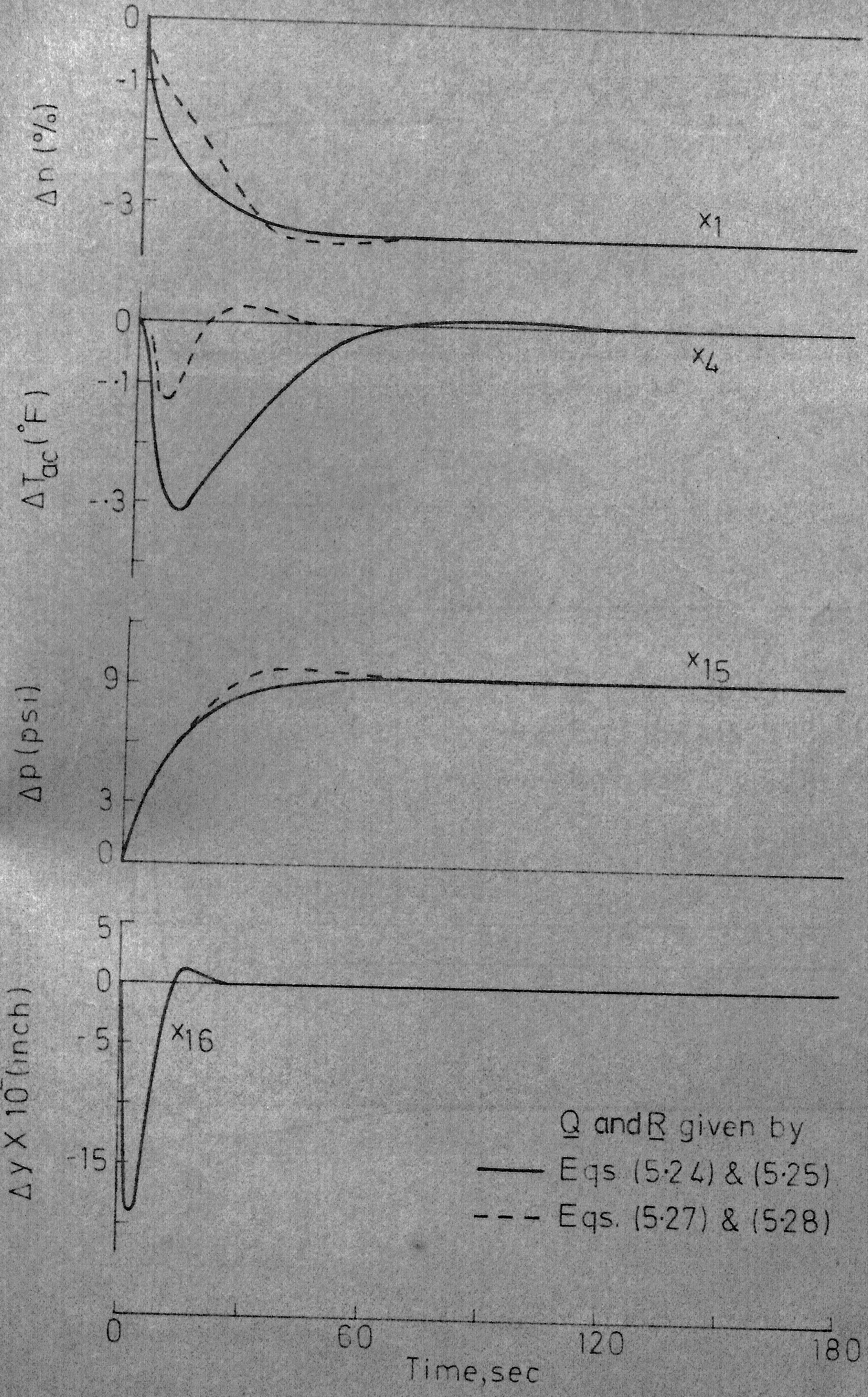


FIG. 5.4 RESPONSE OF LINEAR MODEL WITH PROPORTIONAL AND INTEGRAL STATE FEEDBACK

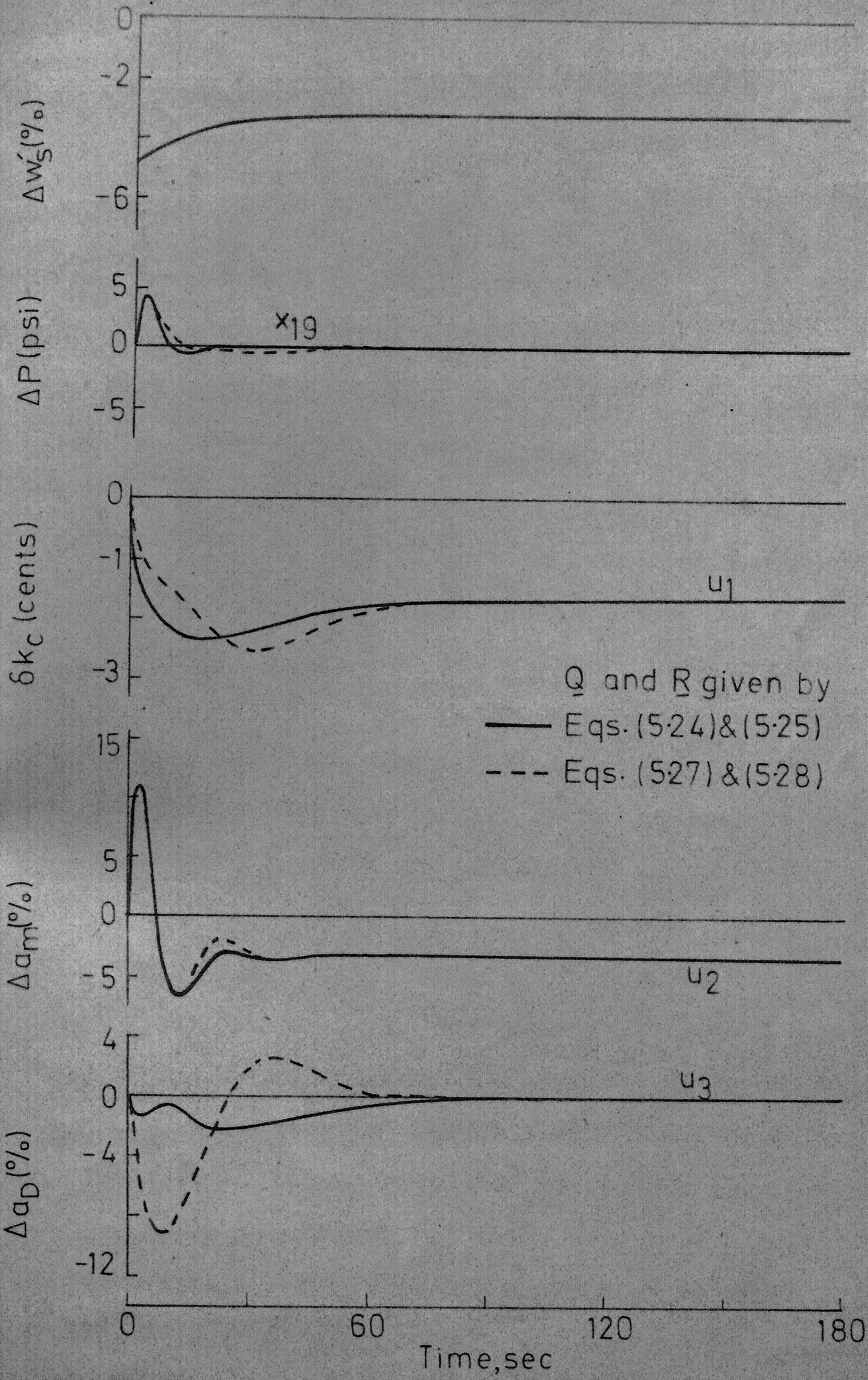


FIG. 5·4 (CONTINUED)

$$\underline{Q} = \text{diag}\ (6, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 100, 0.1, 0.1) \tag{5.27}$$

and the matrix $\underline{R}$ is maintained as in Eq.(5.25), i.e.

$$\underline{R} = \text{diag}\ (1,1,1) \tag{5.28}$$

Optimal control law of the form given by Eq.(5.26) is obtained with these weighting matrices and the response of the system for a -5 percent step in w_1 is shown (as dotted lines) in Figure 5.4. It can be seen that the increase in the weight on coolant temperature has reduced the variations in ΔT_{ac} considerably. The reactivity δk_c has changed marginally when compared with the reactivity needed for the previous weighting matrices. The magnitude of Δa_D has increased. This is due to the high sensitivity of the pressure of the primary heat transport system to the variations of temperature in the circuit.

5.4 CONCLUSIONS

In this chapter, optimal control theory has been applied to the dynamics of the linear model of the pressurised heavy water reactor to obtain feedback controllers. The application of linear regulator theory has resulted in proportional controller and the closed loop behaviour of the reactor with the proportional controller is stable with steady state off-sets in the presence of constant disturbance. These off-sets are eliminated by modifying the performance index.

The modified performance index also yields a proportional controller but it is not feasible to implement this controller since it requires a knowledge of the nature and the magnitude of the step disturbance.

In order to overcome the drawback of the proportional controller, a proportional - integral controller is envisaged and the linear regulator theory is modified to yield the proportional-integral controller and the performance of the system with this controller is obtained for a constant disturbance. Eventhough the response of the system is acceptable, the implementation of the controller requires a knowledge of all the states. However only a few states like the average temperature of the coolant in the reactor, the water level in the steam drum and the pressure of the primary heat transport system are directly measurable and have practical significance. Hence it is pertinent to have a control law which is not necessarily optimal, but is dependent only on the available states. A control of this nature, known as sub-optimal control, is developed in the next chapter.

CHAPTER 6

SUB-OPTIMAL CONTROL

6.1 INTRODUCTION

The concept of state space and the use of Pontryagin maximum principle have resulted, as shown in the previous chapter, in optimal feed back control laws which call for a complete measurement of the state vector. In many physical systems, the state variables need not necessarily correspond to physically measurable quantities. To realise the control law, it is often suggested that the unavailable state variables can be constructed via a Kalman-Bucy filter or an observer. But it is quite impractical to reconstruct the state variables in a large multi-variable system such as those which occur in chemical processes, power systems etc. In these cases it would be convenient if the designer has the freedom to select based on experience or intution certain variables and make the control a function of these state variables. This may result in the deterioration of the performance index; but if the performance of the system is close to the performance of the optimally controlled system, then it would be advantageous to go in for this control law. This type of feedback control scheme where the control variables are linear combinations of selected state variables is obtained in this chapter[45].

$$\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} k_{11} & k_{12} & 0 & 0 \\ 0 & 0 & k_{23} & k_{24} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

The design of the constrained controller then reduces to the selection of values for the non-zero elements of the feedback matrix $\underline{K}$.

6.3 OPTIMAL DESIGN OF CONSTRAINED CONTROLLER

If the constrained feedback matrix $\underline{K}$ is to generate optimal control of the form $\underline{u} = \underline{K}\ \underline{x}$ for the system represented by Eq.(6.2) and the performance index J given by Eq.(6.1), then it is necessary that $\partial J/\partial k_{ij}$ must vanish for each allowed feedback k_{ij}. Define a matrix $\underline{G}$ of order mxn with its element g_{ij} representing the gradient of J with respect to the element

k_{ij} , i.e.,

$$g_{ij} = \frac{\partial J}{\partial k_{ij}}$$

Then for optimality, g_{ij} must be zero for each allowable feedback i and j. Jameson[45] has shown that the gradient matrix $\underline{G}$ can be written as

$$\underline{G} = 2\ (\underline{B}^T\underline{P} + \underline{R}\ \underline{K})\ \underline{W} \tag{6.4}$$

where $\underline{W}$ and $\underline{P}$ satisfy the following equations

$$\underline{F}\ \underline{W} + \underline{W}\ \underline{F}^T + \underline{x}_o\ \underline{x}_o^T = \underline{O} \tag{6.5}$$

$$\underline{F}^T\ \underline{P} + \underline{P}\ \underline{F} + \underline{Q} + \underline{K}^T\ \underline{R}\ \underline{K} = \underline{O} \tag{6.6}$$

$$\text{with } \underline{F} = \underline{A} + \underline{B}\ \underline{K} \tag{6.7}$$

Further the value of the performance index J is given by

$$J = \underline{x}_o^T \underline{P} \underline{x}_o$$

$$= \text{trace } [\underline{P} \underline{x}_o \underline{x}_o^T] \qquad (6.8)$$

Thus Eqs. (6.8) and (6.4) provide explicit expressions for the magnitude of the performance index and its gradients with respect to the feedback matrix $\underline{K}$. The design of optimal constrained controller now reduces to the selection of the allowable feedback elements k_{ij} such that $g_{ij}=0$ for each allowable i and j.

6.3.1 Elimination of Dependency on Initial Conditions

It can be seen from Eqs.(6.5) and (6.8) that the gradient matrix $\underline{G}$ and the value of the performance index J depend on the initial condition $\underline{x}_o$, specified by Eq.(6.2). To overcome the dependency on initial condition, it is generally assumed[46] that $\underline{x}_o$ lies on the surface of a unit sphere in the n dimensional eucledian space and $\underline{x}_o \underline{x}_o^T$ is taken as an identity matrix $\underline{I}$. Substituting $\underline{I}$ for $\underline{x}_o \underline{x}_o^T$ in Eqs.(6.5) and (6.8) the matrix $\underline{G}$ and J can be obtained from the following equations.

a) $\underline{G}$ is given by

$$\underline{G} = 2 (\underline{B}^T\underline{P} + \underline{R} \underline{K}) \underline{W} \qquad (6.9)$$

where $\underline{W}$ and $\underline{P}$ satisfy the following equations

$$\underline{F} \underline{W} + \underline{W} \underline{F}^T + \underline{I} = \underline{0} \qquad (6.10)$$

$$\underline{F}^T \underline{P} + \underline{P} \underline{F} + \underline{Q} + \underline{K}^T \underline{R} \underline{K} = 0 \qquad (6.11)$$

with $\underline{F} = \underline{A} + \underline{B}\,\underline{K}$ (6.12)

b) J is given by

$$J = \text{trace}\,[P] \tag{6.13}$$

Procedure for obtaining the optimal feedback matrix $\underline{K}$ using the above equations is discussed in the following section.

6.3.2 Method of Solution

Having obtained explicit expressions for the performance index J and its gradient with respect to the feedback matrix $\underline{K}$, the procedure of obtaining the optimal controller is stated as: Selecting an initial guess for $\underline{K}$, Eqs.(6.10), (6.11) and (6.12) are solved[41] to obtain the matrices $\underline{P}$ and $\underline{W}$. These matrices are then used in Eqs.(6.13) and (6.9) to calculate the performance index J and its gradient matrix $\underline{G}$. Since the problem of designing the optimal controller is one of obtaining the elements k_{ij} for each allowable i and j, which will result in the corresponding $g_{ij} = 0$, the problem is treated as one of minimising a function using its gradients. The function thus becomes the performance index J and the gradient of the function with respect to each variable, i.e., allowable feedback k_{ij}, is provided in the matrix $\underline{G}$. A descent technique based on the gradients such as Fletcher-Reeves (given in Appendix F) is now used to minimise J and to improve the feedback matrix $\underline{K}$. This $\underline{K}$

matrix is then used to start the procedure and the procedure is repeated till the condition for optimality is achieved i.e., $g_{ij} = 0$ for each allowable i and j.

6.4 DESIGN OF CONSTRAINED CONTROLLER FOR THE PRESSURISED HEAVY WATER REACTOR

The method described in the preceeding section is now applied to the pressurised heavy water reactor to develop an optimal controller of specified configuration. The reactor, the dynamics of which is represented by Eq.(4.7), has three control variables. These are the reactivity u_1, the area of the feedwater valve u_2 and the area of the bleed or feed valve for the primary circuit heavy water u_3. State variables of primary interest for efficient operation of the reactor are the following

i) average temperature of the coolant in the reactor x_4

ii) water level in the steam drum x_{16}

iii) pressure of the primary coolant circuit x_{19} and

iv) steam drum pressure x_{15}

The design and operation of the reactor requires that the first three of the above four variables are to be held at their initial values. These variables are easily measurable and are readily available for control purposes. Therefore the three control variables must be generated from these four variables. To achieve this, the following control

configuration, which is similar to conventional analog controller is postulated.

$$u_1 = a_1 x_4 + a_2 \int_0^t x_4 \, ds \tag{6.14}$$

$$u_2 = b_1 x_{16} + b_2 \int_0^t x_{16} ds \tag{6.15}$$

$$u_3 = c_1 x_{19} \tag{6.16}$$

It is now required to find optimal values for the coefficients a_1, a_2, b_1, b_2 and c_1.

The controller represented by Eqs.(6.14), (6.15) and (6.16) is of the proportional - integral type. In Chapter 5, the proportional-integral controller has been designed by modifying the linear regulator theory. Following the method outlined in that chapter, the dynamics of the pressurised heavy water reactor can be represented by the equations (5.16), (5.18) and (5.19) which are repeated below for convenience

Reactor dynamics

$$\underline{\dot{z}} = \underline{\tilde{A}}\ \underline{z} + \underline{\tilde{B}}\ \underline{u} \tag{5.16}$$

$$\underline{\tilde{A}} = \begin{bmatrix} \underline{A} & \underline{O} \\ \underline{C} & \underline{O} \end{bmatrix} ; \quad \underline{\tilde{B}} = \begin{bmatrix} \underline{B}_1 \\ \underline{D}_1 \end{bmatrix}$$

where $\underline{z}^T = (\underline{\dot{x}}^T, \underline{y}^T)$

$$\underline{y}^T = (x_4, x_{16}, x_{19})$$

and $\underline{v} = \underline{\dot{u}}$

Performance Index: $J = \int_0^\infty (\underline{z}^T \underline{Q}\, \underline{z} + \underline{v}^T \underline{R}\, \underline{v})\, dt$ (5.18)

and the control law is $\underline{v} = \underline{K}\,\underline{z}$ (5.19)

The proposed controller, represented by Eqs.(6.14), (6.15) and (6.16) is transformed into the system variables $\underline{z}$ and $\underline{v}$ and the control variables can be expressed as

$$
\begin{aligned}
\dot{u}_1 = v_1 &= a_1 \dot{x}_4 + a_2 x_4 && = a_1 \dot{x}_4 + a_2 y_1 \\
& && = a_1 z_4 + a_2 z_{20} \\
\dot{u}_2 = v_2 &= b_1 \dot{x}_{16} + b_2 x_{16} && = b_1 \dot{x}_{16} + b_2 y_2 \\
& && = b_1 z_{16} + b_2 z_{21} \\
\dot{u}_3 = v_3 &= c_1 \dot{x}_{19} && = c_1 z_{19}
\end{aligned}
$$

Therefore, the proposed controller when expressed as $\underline{v} = \underline{K}\,\underline{z}$ has a specified structure with the elements $k_{1,4}$, $k_{1,20}$, $k_{2,16}$, $k_{2,21}$, $k_{3,19}$ being allowed to take non-zero values.

The weighting matrices $\underline{Q}$ and $\underline{R}$ for the performance index in Eq.(5.18) are chosen as

$$\underline{Q} = \text{diag}\,(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0.1, 0.1) \quad (6.17)$$

and $$\underline{R} = \text{diag}\,(1,1,1) \quad (6.18)$$

In order to select the optimal values for the non-zero elements of the feed back matrix, the procedure discussed in Section 6.3 is now applied to the system represented by Eqs.(5.16) and (5.18) with the weighting matrices given by Eqs.(6.17) and (6.18). The minimisation of performance index J is done with the use

of Fletcher-Reeves method (given in Appendix F). and the optimum values obtained for the non-zero elements of the feedback matrix $\underline{F}$ are given by

$$k_{1,4} = -1.114 \qquad k_{1,20} = -0.7997$$
$$k_{2,16} = -1.0533 \qquad k_{2,21} = -0.00926$$
$$\text{and } k_{3,19} = -19.985$$

Since $k_{1,4}$, $k_{1,20}$, $k_{2,16}$, $k_{2,21}$, $k_{3,19}$ correspond to a_1, a_2, b_1, b_2, c_1 respectively, the control scheme specified by Eqs.(6.14), (6.15) and (6.16) can be written as

$$u_1 = -1.114 \ x_4 - 0.7997 \int_0^t x_4 \, ds \tag{6.19}$$

$$u_2 = -1.0533 \ x_{16} - 0.00926 \int_0^t x_{16} ds \tag{6.20}$$

$$u_3 = -19.985 \ x_{19} \tag{6.21}$$

These equations constitute the optimal controller of the specified structure.

The performance of the reactor with this controller providing the feedback control is obtained for a -5 percent step change in the area of the steam valve (Fig.6.1). The dynamic behaviour of the system with this control law compares well with the behaviour of the system with complete state feedback (Fig.5.4). However it can be seen that the transient responses of the average temperature of the coolant in the reactor ΔT_{ac} are different for the two control laws. Also

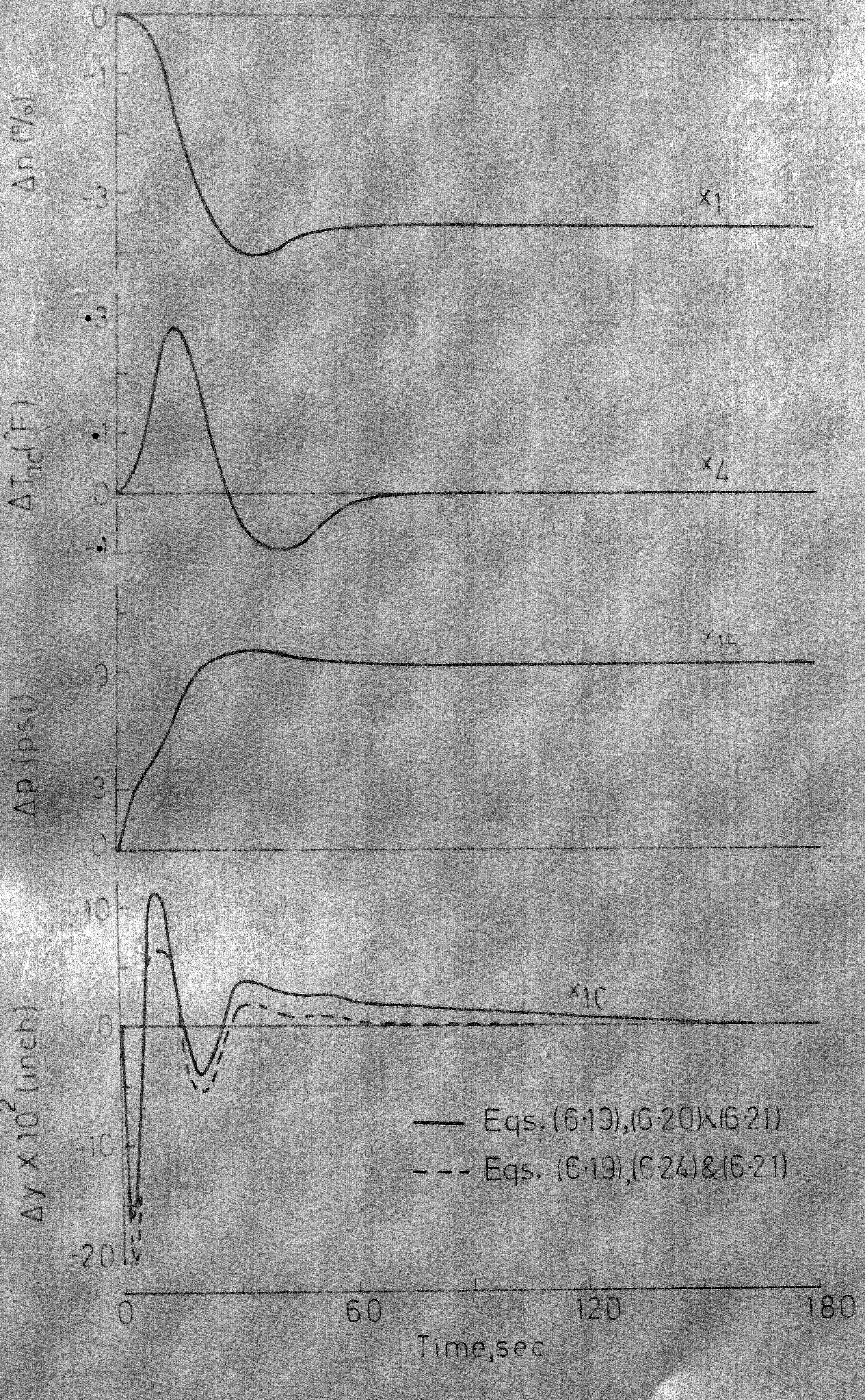


FIG.6.1 RESPONSE OF LINEAR MODEL WITH OPTIMAL ANALOG CONTROLLER

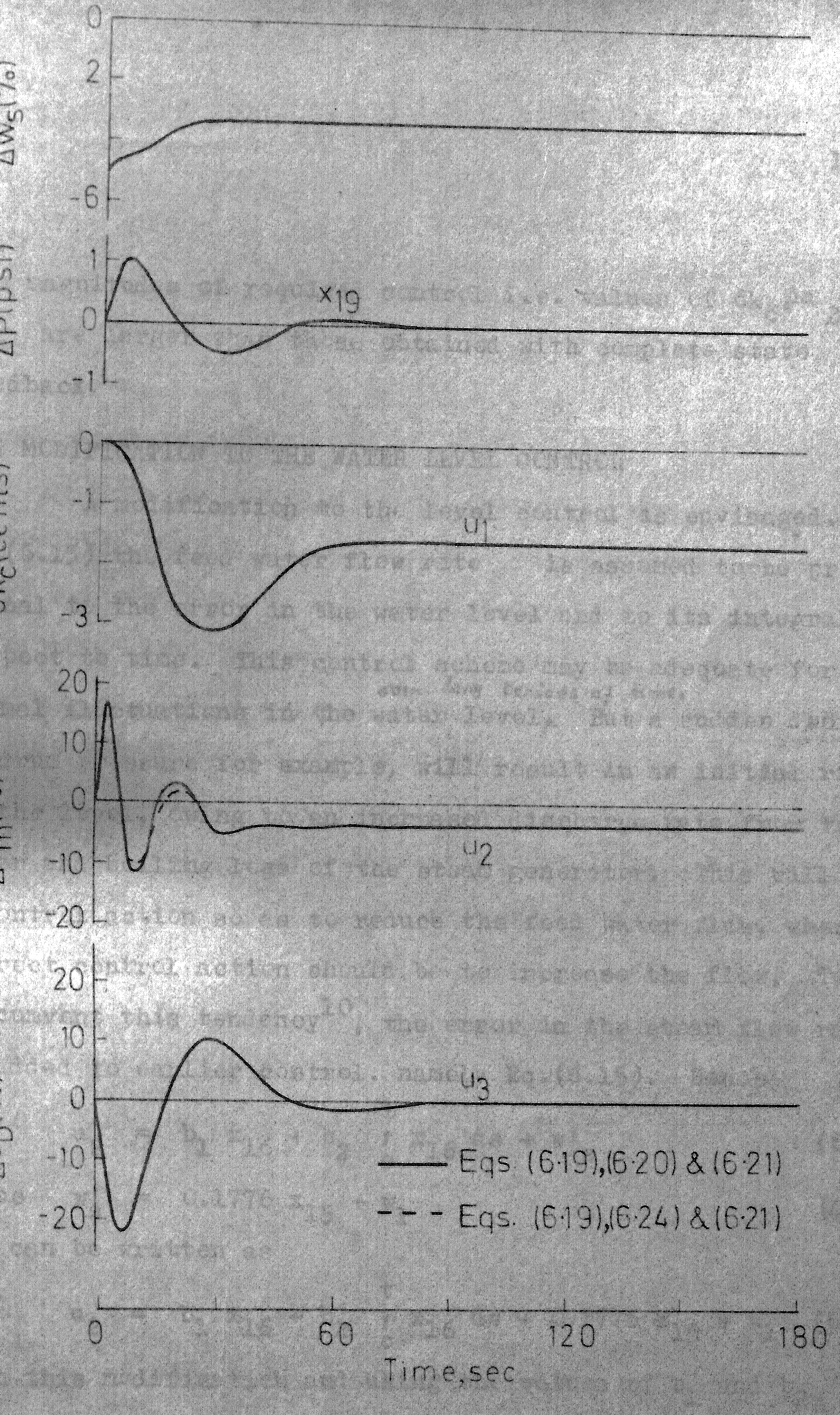


FIG. 6·1 (CONTINUED)

the magnitudes of required control i.e. values of δk_c, Δa_D and Δa_m are larger than those obtained with complete state feedback.

6.5 MODIFICATION TO THE WATER LEVEL CONTROL

A modification to the level control is envisaged. In Eq.(6.15) the feed water flow rate is assumed to be proportional to the error in the water level and to its integral with respect to time. This control scheme may be adequate for normal fluctuations in the water level over long periods of time. But a sudden decrease in drum pressure for example, will result in an initial rise in the level, owing to an increased discharge rate from the riser and boiling legs of the steam generator. This will initiate a control action so as to reduce the feed water flow, whereas the correct control action should be to increase the flow. To circumvent this tendency[10], the error in the steam flow rate is added to earlier control, namely Eq.(6.15). Hence

$$u_2 = b_1 x_{16} + b_2 \int_0^t x_{16}\, ds + w'_s \tag{6.22}$$

Since $w'_s = 0.1776\, x_{15} + w_1$ (4.5)

u_2 can be written as

$$u_2 = b_1 x_{16} + b_2 \int_0^t x_{16}\, ds + 0.1776\, x_{15} + w_1 \tag{6.23}$$

With this modification and using the values of b_1 and b_2 obtained from Sec.(6.4), u_2 is written as

$$u_2 = -1.0533\, x_{16} - 0.00926 \int_0^t x_{16}\, ds + 0.1776\, x_{15} + w_1 \tag{6.24}$$

Closed loop responses of the system, for a -5 percent step change in the area of steam valve w_1, with the control action provided by Eqs.(6.19),(6.21) and (6.24) are obtained and are shown as dotted lines in Fig.6.1. It is seen that the above modification has not reduced the erroneous initial movement of the feed water control valve u_2. This may be due to the heavy weightage provided in the matrix $\underline{Q}$ for the level. It is expected that a reduction in the weighting factor for level, namely $q_{21,21}$, will allow variations in the level and will reduce the unnecessary movement of the feed water valve. The weighting factor $q_{21,21}$ is to be suitably selected such that the initial movement of the control valve is governed by the steam flow rate and the level error signal is used to maintain the drum level at the correct value over long periods of time.

6.6 PERFORMANCE OF THE NONLINEAR MODEL OF THE PHWR

In Sec.6.5, a controller of specified structure which is quite similar to an analog controller is designed by applying the optimal control theory to the linear model of the pressurised heavy water reactor. In order to study the behaviour of the plant with this controller, the controller is incorporated in the dynamics of the plant which is represented by the nonlinear model (of chapter 3). The performance of the system for a -5 percent step in the area of steam valve is obtained and is now discussed.

6.6.1 Controlled Response for a -5 percent step in Area of Steam Valve

The controller obtained in Sec.6.5 is represented by Eqs.(6.19), (6.21) and (6.24). These equations are repeated below for convenience.

$$u_1 = -1.114 \; x_4 - 0.7997 \int_0^t x_4 \, ds \qquad (6.19)$$

$$u_2 = -1.0533 \; x_{16} - 0.00926 \int_0^t x_{16} \, ds + 0.1776 \; x_{15} + w_1 \qquad (6.24)$$

$$u_3 = -19.985 \; x_{19} \qquad (6.21)$$

Substituting the physical plant variables for the state and control variables, these equations can be written as

$$u_1 = \delta k_c = -1.114 \; \Delta T_{ac} - 0.7997 \int_0^t \Delta T_{ac} \, ds \qquad (6.25)$$

$$u_2 = \Delta a_m = -1.0533 \; \Delta y - 0.00926 \int_0^t \Delta y \, ds + 0.1776 \; \Delta p + \Delta w_1 \qquad (6.26)$$

$$u_3 = \Delta a_D = -19.985 \; \Delta P \qquad (6.27)$$

Since Δx represents the deviation of the variable x from the initial steady state value, Δx can be written as

$$\Delta x = x(t) - x(o)$$

Further Δa_m and Δa_D represent the deviations as percentages; therefore Δa_m can be written as

$$\Delta a_m = [a_m(t) - a_m(o)] \times 100.0$$

Hence the actual valve opening $a_m(t)$ becomes

$$a_m(t) = a_m(o) + \Delta a_m \times 0.01$$

Similarly $a_D(t) = a_D(o) + \Delta a_D \times 0.01$

and $a_s(t) = a_s(o) + \Delta w_1 \times 0.01$

The initial values of theplant variables are given below

$$T_{ac}(0) = 520.68^{\circ}F$$

$$y(0) = 0.0 \text{ ft}$$

$$p(0) = 568.9 \text{ psia}$$

$$P(0) = 1250.3 \text{ psia}$$

$$a_m(0) = 1.0$$

$$a_D(0) = 0.0$$

$$a_s(0) = 1.0$$

In the linear model the level y has been expressed in the unit of 10^{-2} inch where as in the nonlinear model it has been expressed in feet. Making appropriate corrections in y for the change of units between the two models and substituting the initial values for plant variables, the controller given by Eqs.(6.25), (6.26) and (6.27) can be represented as

$$\delta k_c = -1.114\,\varepsilon - 0.7997 \int_0^t \varepsilon \, ds \tag{6.28}$$

where $\varepsilon = T_{ac}(t) - 520.68$

$$a_m(t) = 12 \times (-1.0553\, y - 0.00926 \int_0^t y \, ds) + 0.001776\,(p-568.9) + a_s \tag{6.29}$$

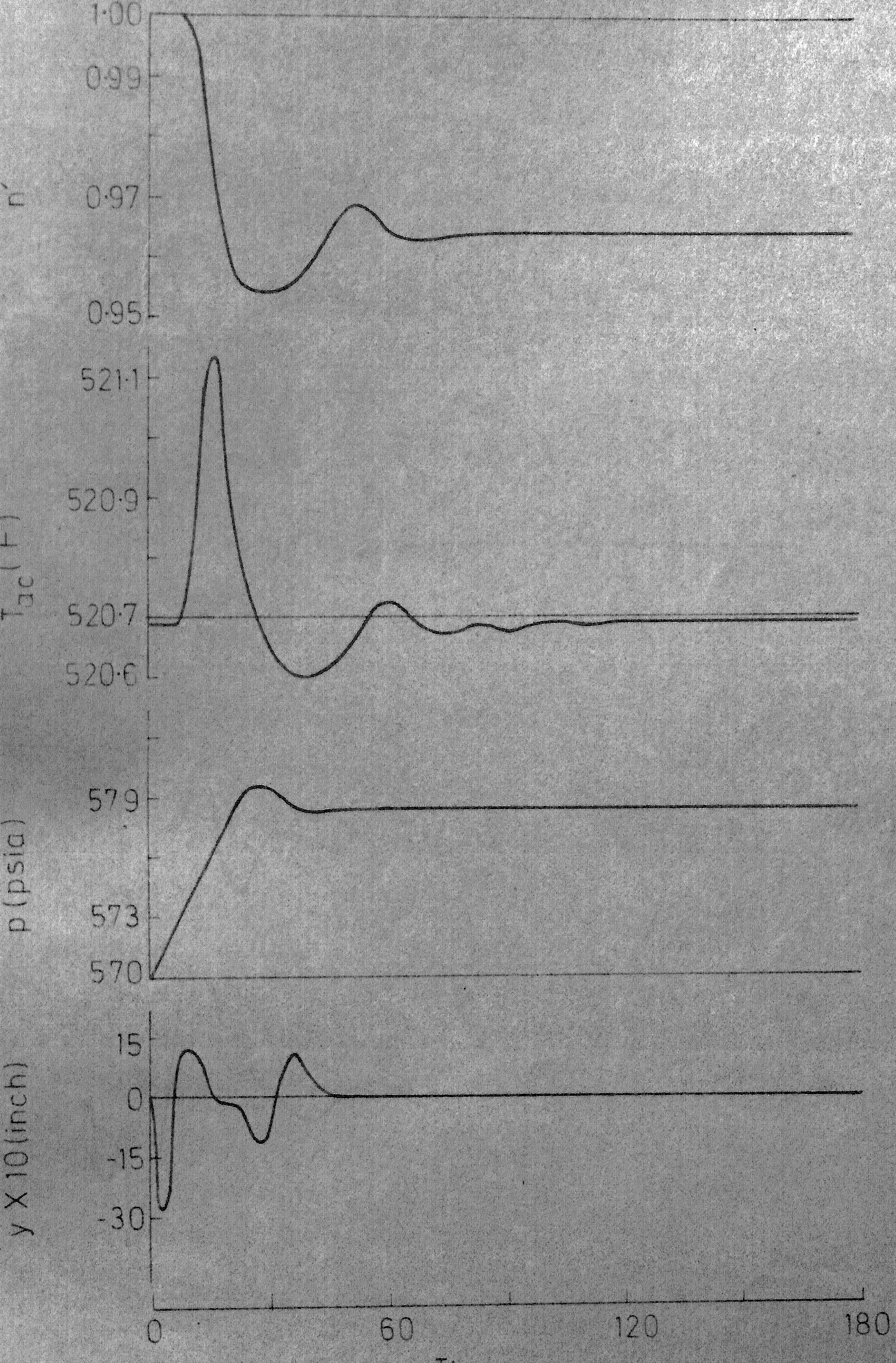
$$a_D(t) = -0.19985\ (P-1250.3) \qquad (6.30)$$

Thus Eqs.(6.28), (6.29) and (6.30) represent the analog controller scheme, as applicable to the non-linear model.

This analog controller is included to the dynamics of the nonlinear model which is represented by Eq.(3.17). The response of the system for a -5 percent step in the area of steam valve i.e. $a_s = 0.95$ is obtained (Fig.6.2). It can be seen from these responses that there is no change in the values of neutron power n' and the average temperature of the coolant in the reactor T_{ac} for the first seven seconds. This is due to the transport time lag of the coolant between the steam generator and the reactor. At the end of 7 seconds, T_{ac} increases sharply due to the increase in the inlet temperature of the coolant to the reactor; this introduces a steep decrease in the reactivity δk_c and steep variations in the area of the valve opening a_D.

6.6.2 Improvement of the Responses by the Feedback of Steam Drum Pressure

The responses obtained in the preceeding section can however be improved by modifying the reactivity control. The sharp changes in T_{ac}, a_D and δk_c are mainly due to the transport delay between the steam generator and the reactor. It is expected that the effect of the transport delay can be reduced by including steam drum pressure as an anticipatory



FIG·62 RESPONSE OF NONLINEAR MODEL WITH OPTIMAL ANALOG CONTROLLER (WITHOUT PRESSURE FEEDBACK)

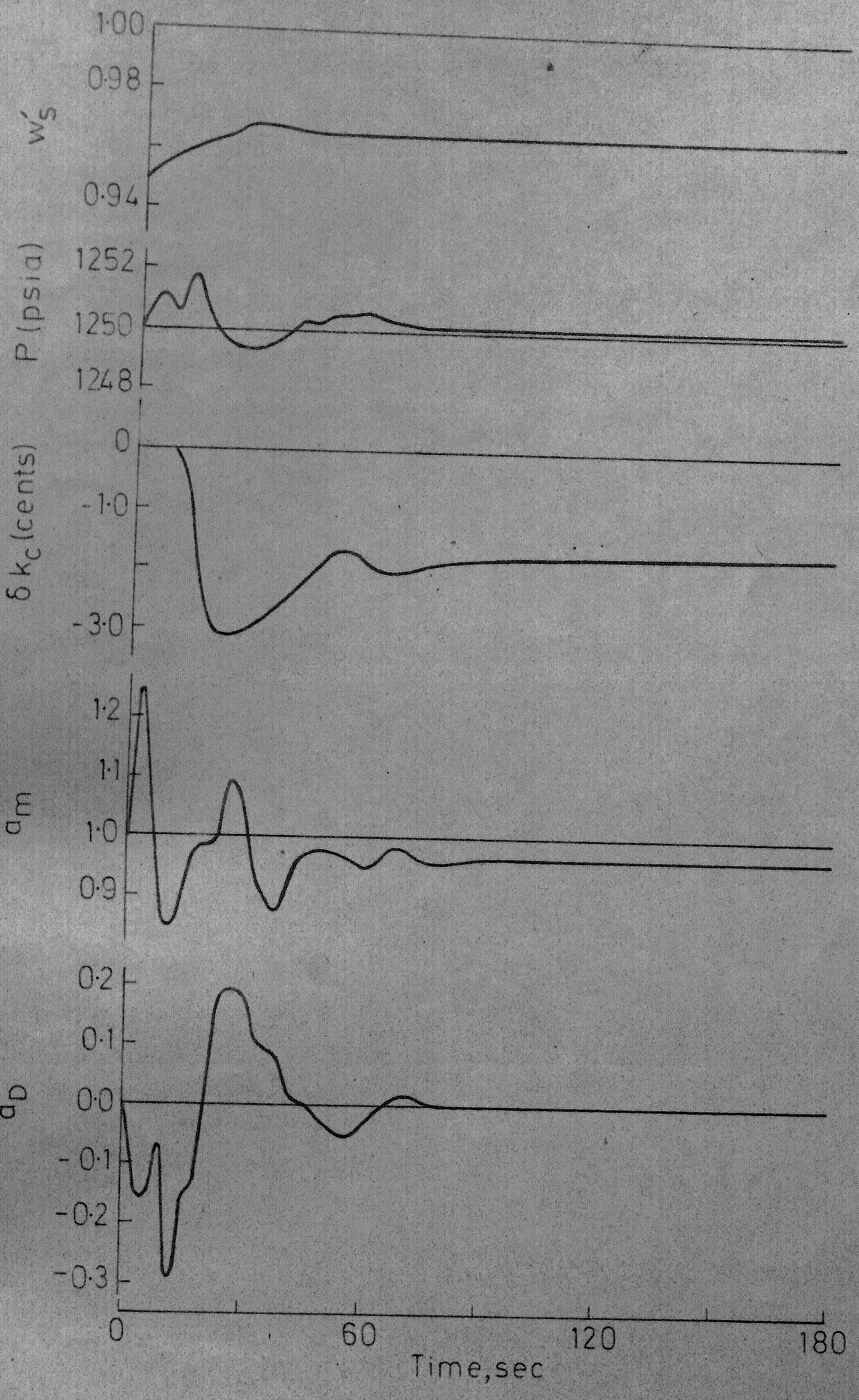


FIG. 6·2 (CONTINUED)

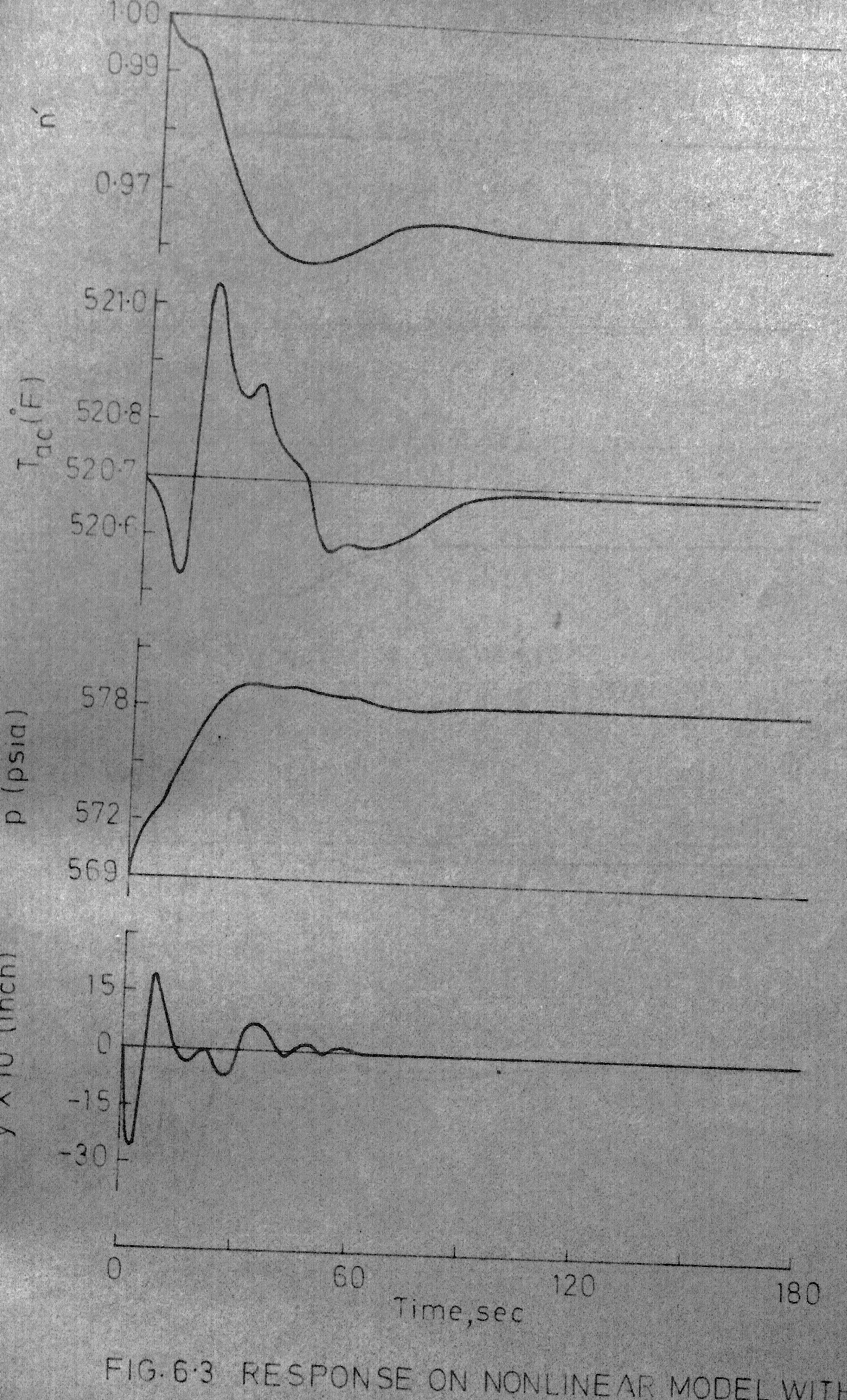


FIG. 6.3 RESPONSE ON NONLINEAR MODEL WITH OPTIMAL ANALOG CONTROLLER (WITH PRESSURE FEEDBACK)

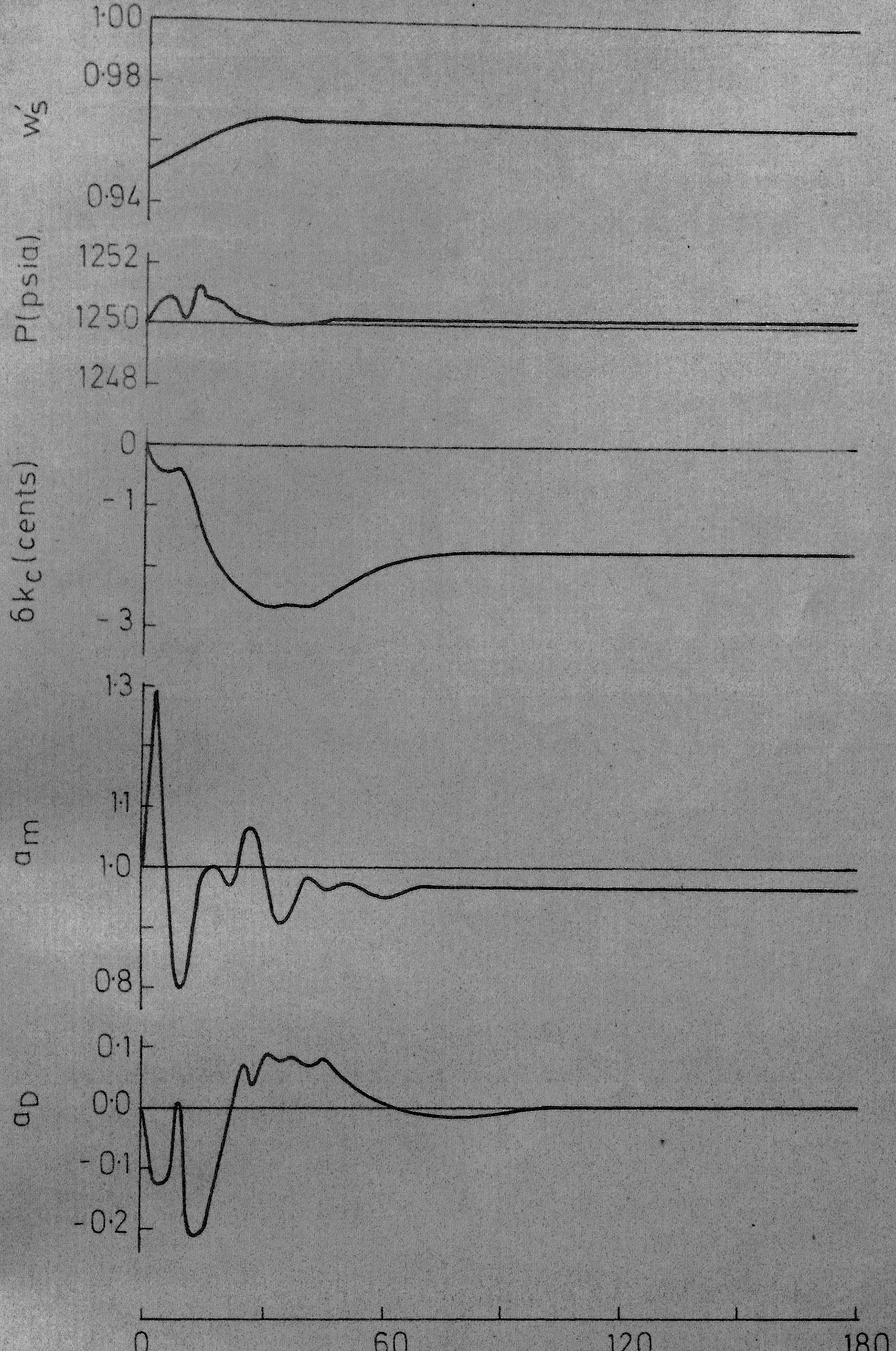


FIG. 6·3 (CONTINUED)

signal in the reactivity control law Eq.(6.28).

The control law for the linear model incorporating steam drum pressure in the reactivity control can be written as

$$u_1 = \delta k_c = a_1 \Delta T_{ac} + a_2 \int_0^t \Delta T_{ac}\, ds + a_3 \Delta p \qquad (6.31)$$

$$u_2 = \Delta a_m = b_1 \Delta y + b_2 \int_0^t \Delta y\, ds \qquad (6.32)$$

$$u_3 = \Delta a_D = c_1 \Delta P \qquad (6.33)$$

Optimal values for the constants a_1, a_2, a_3, b_1, b_2 and c_1 are obtained for this controller by the method discussed in Section 6.4. The cost matrices $\underline{Q}$ and $\underline{R}$ are the same as given by Eqs.(6.17) and (6.18). The optimum values thus obtained are

$$a_1 = -1.383 \quad a_2 = -0.2263 \quad a_3 = -0.17245$$

$$b_1 = -1.1829 \quad b_2 = -0.08803$$

$$\text{and} \quad c_1 = -25.7$$

Therefore the control law for the nonlinear model can be written, similar to Eqs.(6.28), (6.29) and (6.30), as

$$\delta k_c = -1.383\ \varepsilon - 0.2263 \int_0^t \varepsilon\, ds - 0.17245\ (p-568.9) \qquad (6.34)$$

$$a_m(t) = 12.0 \times (-1.1829\ y - 0.08803 \int_0^t y\, ds)$$

$$+ 0.001776\ (p-568.9) + a_s \qquad (6.35)$$

$$a_D(t) = -0.257\ (P-1250.3) \qquad (6.36)$$

The response of the system with this control scheme for a -5 percent step in the area of the steam value is

signal in the reactivity control law Eq.(6.28).

The control law for the linear model incorporating steam drum pressure in the reactivity control can be written as

$$u_1 = \delta k_c = a_1 \Delta T_{ac} + a_2 \int_0^t \Delta T_{ac}\, ds + a_3 \Delta P \qquad (6.31)$$

$$u_2 = \Delta a_m = b_1 \Delta y + b_2 \int_0^t \Delta y\, ds \qquad (6.32)$$

$$u_3 = \Delta a_D = c_1 \Delta P \qquad (6.33)$$

Optimal values for the constants a_1, a_2, a_3, b_1, b_2 and c_1 are obtained for this controller by the method discussed in Section 6.4. The cost matrices $\underline{Q}$ and $\underline{R}$ are the same as given by Eqs.(6.17) and (6.18). The optimum values thus obtained are

$$a_1 = -1.383 \quad a_2 = -0.2263 \quad a_3 = -0.17245$$

$$b_1 = -1.1829 \quad b_2 = -0.08803$$

and $c_1 = -25.7$

Therefore the control law for the nonlinear model can be written, similar to Eqs.(6.28), (6.29) and (6.30), as

$$\delta k_c = -1.383\ \varepsilon - 0.2263 \int_0^t \varepsilon\, ds - 0.17245\ (p-568.9) \qquad (6.34)$$

$$a_m(t) = 12.0 \times (-1.1829\ y - 0.08803 \int_0^t y\, ds) + 0.001776\ (p-568.9) + a_s \qquad (6.35)$$

$$a_D(t) = -0.257\ (P-1250.3) \qquad (6.36)$$

The response of the system with this control scheme for a -5 percent step in the area of the steam value is

shown in Fig.6.3. The performance of the system is better than that of the system with the previous control scheme and control input a_D is also less.

6.7 CONCLUSIONS

In this chapter, optimal analog controllers are obtained for the linear model of the pressurised heavy water reactor and the dynamic behaviour of the linear model is studied. Later these controllers are suitably modified and incorporated in the nonlinear model and the performance of the reactor for a -5 percent step change in the area of steam valve is analysed.

CHAPTER 7

SUMMARY AND SUGGESTIONS

7.1 SUMMARY AND CONCLUSIONS

A major portion of the present day energy requirements is met by hydroelectric and fossil fuelled thermal power stations. However the increasing demand for power and the current oil crisis are bound to place more emphasis on nuclear power stations. This will require development of control system that will increase plant availability and prevent outages; that is, there is a need to design control schemes that will result in efficient and safe operation of the plant. With this aim, controllers are designed for a pressurised heavy water reactor.

The first step to design the controller is to understand the process that is to be controlled. Hence a mathematical model of the pressurised heavy water reactor is formulated from the conservation of mass, momentum and energy. The resulting nonlinear model is simulated on an IBM 7044 digital computer to obtain the uncontrolled responses for a variety of disturbances. In order to facilitate the design of controllers, the nonlinear model is linearised around steady state operating conditions and the linear model is subsequently used for the design of the controller.

Since there is a tendency to incorporate process computers in the operation of power plants, it is feasible to use the computers for control purposes also. As a step towards this form of control, feedback control schemes can be designed based on modern control theory. Hence in Chapter 5, optimal control theory is made use of in designing proportional and proportional-integral controllers for the linearised model of the plant. The behaviour of the system with these controllers are discussed and the drawbacks of these controllers are also outlined.

Power stations in general have analog controllers. Therefore it would be advantageous to adjust the gains of the existing controllers or to modify the control configurations so as to improve the performance of the plant, without resorting to new control schemes (of chapter 5). Therefore in Chapter 6, optimal values for the gains of an analog proportional-integral controller is obtained by using linear regulator theory and function minimisation techniques. This controller has been designed based on the linear model. Therefore it is modified to include the initial operating conditions of the reactor. Then it is incorporated in the nonlinear model of the plant. The response of the plant with this controller is obtained and is found to be satisfactory for the disturbance studied.

7.2 SUGGESTIONS FOR FURTHER STUDY

Some of the related problems for further study are:

i) The fluctuations in the feedwater valve may be eliminated or reduced by decreasing the weightage on the level variation or by increasing the weightage on valve movements. This will allow for fluctuations in the level during the early part of the transients with the primary control of level being effected by steam flow rate and with the proportional-integral feedback of level acting as a readjusting control so as to maintain the drum level at the correct value over long periods of time.

ii) a) In the analysis, the dynamics of the various valves such as steam valve, feed water valve and bleed or feed valve for primary coolant may be considered. This will yield a true representation of the system.

b) Further it is realistic to consider the rate of change of reactivity, i.e., $\frac{d}{dt}\,\delta k_c$ as the control input rather than δk_c itself. The analysis then will include the behaviour of the control-rod drive mechanism.

c) In this study, feed water flow has been assumed to depend only on the area of the valve opening. This can be modified as to incorporate the dynamics of the boiler feed pump and its dependence on the steam drum pressure.

iii) The method of designing optimal analog controller can be extended

a) to adjust the gain of the existing analog controllers and

b) to achieve a controller that will make the reactor follow the turbine load fluctuations.

iv) Since the primary heat transport system is sensitive to energy transfer from the reactor to the steam generator, the pressure of this system can be used to provide an anticipatory control; that is, the primary circuit pressure can be utilised to vary the external reactivity. This procedure will reduce the dependency of reactivity control on the transport delay between the reactor and the steam generator.

v) Lastly, the dynamics of turbine and generator can be included in the model to provide a complete understanding of the power station.

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APPENDIX A

FUEL DYNAMICS

Major portion of the heat is released within the fuel itself. This heat is transferred to the coolant through heat conduction in the fuel and the canning (clad). The heat transfer from the fuel to the canning is a mixture of convection and direct metallic contact.

The following assumptions are made in evaluating the average temperature of the fuel.

1. All fuel rods in the core are assumed to have the same power. Power generation in the fuel rod is assumed to be constant (i.e. independent of radius).
2. Temperature in the fuel depends only on radial distance from the centre line of the fuel rod.
3. Thermal conductivity of fuel is described by+

$$k = a/(b+T)$$

4. The heat transfer from fuel surface to outer canning surface is described by one heat transfer coefficient.

To evaluate++ the temperature distribution in the fuel rod, the fuel rod is divided into a number (m) of concentric cylindrical shells, each of equal volume. The radius of inner

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+ I.Devold, 'A Study of the Temperature Distribution in UO_2 Reactor Fuel Elements', Report AE-318, Aktiebolaget Atomenergi, 1968.

++ Power System Simulator - Description of Models, IBM Manual, GE 20-0387-0.

boundary, r_i of shell no. i is then

$$r_i = r_f \left(\frac{i-1}{m}\right)^{\frac{1}{2}}, i = 1,\ 2,\ \ldots\ m$$

where r_f = radius of fuel rod.

Let T_i be the average temperature in shell number i. Then the temperature gradient on the inner boundary of shell i can be written as

$$\left.\frac{dT}{dr}\right|_{r=r_i} = \frac{T_i - T_{i-1}}{\frac{1}{2}(r_{i+1} - r_{i-1})} \qquad i = 2,3,\ \ldots\ ,m$$

$$= 0 \text{ at } i = 1$$

The heat balance in shell no. i, then, is

$$\rho c \frac{\pi r_f}{m} \ell \frac{dT_i}{dt} = \frac{Q}{m} + 2\pi k_{i-1}\ \ell\ r_i \frac{T_{i-1} - T_i}{\frac{1}{2}(r_{i+1} - r_{i-1})}$$

$$- 2\pi k_i\ \ell\ r_{i+1} \frac{T_i - T_{i+1}}{\frac{1}{2}(r_{i+2} - r_i)} \quad i = 2,3,\ldots,m-1$$

where Q = heat generated per fuel rod

ℓ = length of fuel rod

k_i = thermal conductivity of shell number i

ρ = density and

c = specific heat of fuel

The above equation can be written as

$$\frac{dT_i}{dt} \frac{1}{H_f} \left[\frac{Q}{m} + A_f\ (k_{i-1}\ b_{i-1}(T_{i-1} - T_i) - k_i b_i\ (T_i - T_{i+1})) \right]$$

$$i = 1,2\ldots,m-1$$

where

$$H_f = c\,\pi\, r_f^2 \ell/m$$

$$A_f = 4\pi r_f$$

$$b_i = \frac{\sqrt{i}}{\sqrt{i+1} - \sqrt{i-1}} \qquad i = 1,2\ldots,m-1$$

$$b_o = 0$$

For the outer most shell

$$\frac{dT_m}{dt} = \frac{1}{H_f}\left[\frac{Q}{m} + A_f\, k_{m-1}\, b_{m-1}\,(T_{m-1} - T_m) - \frac{A_f}{2}\, h_{gap}(T_{fs}-T_{co})\right] \tag{A.2}$$

where h_{gap} - heat transfer coefficient between the fuel surface and clad outer surface

T_{fs} - temperature at fuel surface

T_{co} - temperature at clad outer surface

On the boundary we have the following equation

$$(T_{fs}-T_{co})\, h_{gap} = k_m \frac{T_m - T_{m+1}}{\frac{r_f}{2}\left(\sqrt{\frac{m+1}{m}} - \sqrt{\frac{m-1}{m}}\right)} \tag{A.3}$$

Here T_{m+1} is the temperature in an auxiliary shell outside the fuel.

Also we have,

$$T_{fs} = T_m - (T_m - T_{m+1})\,\frac{\sqrt{m} - \sqrt{m-1}}{\sqrt{m+1} - \sqrt{m-1}} \tag{A.4}$$

From Eqs. (A.3) and (A.4), T_{fs} is solved and introduced in Eq. (A.2). This leads to

$$\frac{dT_m}{dt} = \frac{1}{H_f}\left[\frac{Q}{m} + A_f\, k_{m-1}\, b_{m-1}\,(T_{m-1} - T_m)\right.$$

$$- \frac{A_f\, h_{gap}\, k_{m-1}\, (T_m - T_{co})}{h_{gap}(1-\sqrt{\frac{m-1}{m}}) + \frac{2k_{m-1}}{r_f}} \Big] \qquad (A.5)$$

Equations (A.1) and (A.5) are used to calculate T_i, $i=1,2,..m$ with a constant value for T_{co}. The average temperature of the fuel T_{af}, can be evaluated as

$$T_{af} = \frac{1}{m} \sum_{i=1}^{m} T_i \qquad (A.6)$$

To calculate the fuel temperature distribution the number of sections (m) was increased until no significant improvement was observed in the calculated values. A value of 6 was used for m and h_{gap} was adjusted so that the average temperature matches with the design value at full power.

Data used:

h_{gap} = 370 W/ft.^{2} ^oF

k = 46x16.9/(480 +(5/9)(T-32)), T< 2500^oF

= 16.9x0.024   T > 2500 ^oF

k in W/ft. ^oF and T in ^oF

r_f = 0.0235 ft.

ρ = 660 lb/ft^3

c = 59.2 WS/lb ^oF

Q = 7000 W/ft. of fuel rod.

T_{co} = 537 ^oF

At full power T_{af} = 1075 ^oF (calculated based on 6 segments)

= 1110 ^oF (design value)

From the steady state full power operation, the temperature response for a +10 % step in power is obtained. The response of the average temperature is shown in Fig.A.1.

Simplified fuel model:

The above representation of fuel dynamics requires m ordinary differential equations with a low limit for integration step about 0.015 sec. A much simpler representation of the fuel dynamics is therefore required. Here the average fuel temperature, T_{af}, is calculated from the following equation

$$\tau_f \frac{dT_{af}}{dt} = \frac{Q}{A_h \; H_f} - (T_{af} - T_{co}) \tag{A.7}$$

where A_h = Heat transfer area at clad outer surface

= $2\pi\ell$x outer radius of clad.

Here H_f is an artificial heat transfer coefficient which is assumed to be a constant and is calculated with the condition that Eqs. (A.6) and (A.7) give the same value for T_{af} at full power steady state operating conditions. Hence H_f becomes equal to 83 W/ft^2 ·F. τ_f was adjusted such that the response of the simplified model compares favourably with that of detailed analysis for a + 10% step in power. The responses are shown in Fig. A.1 with τ_f=5.0 seconds.

Figure A.1 shows that the two responses agree well during the early part of the transients, later settling to different steady state values. This is because, in the detailed analysis the thermal conductivity is treated as a function of

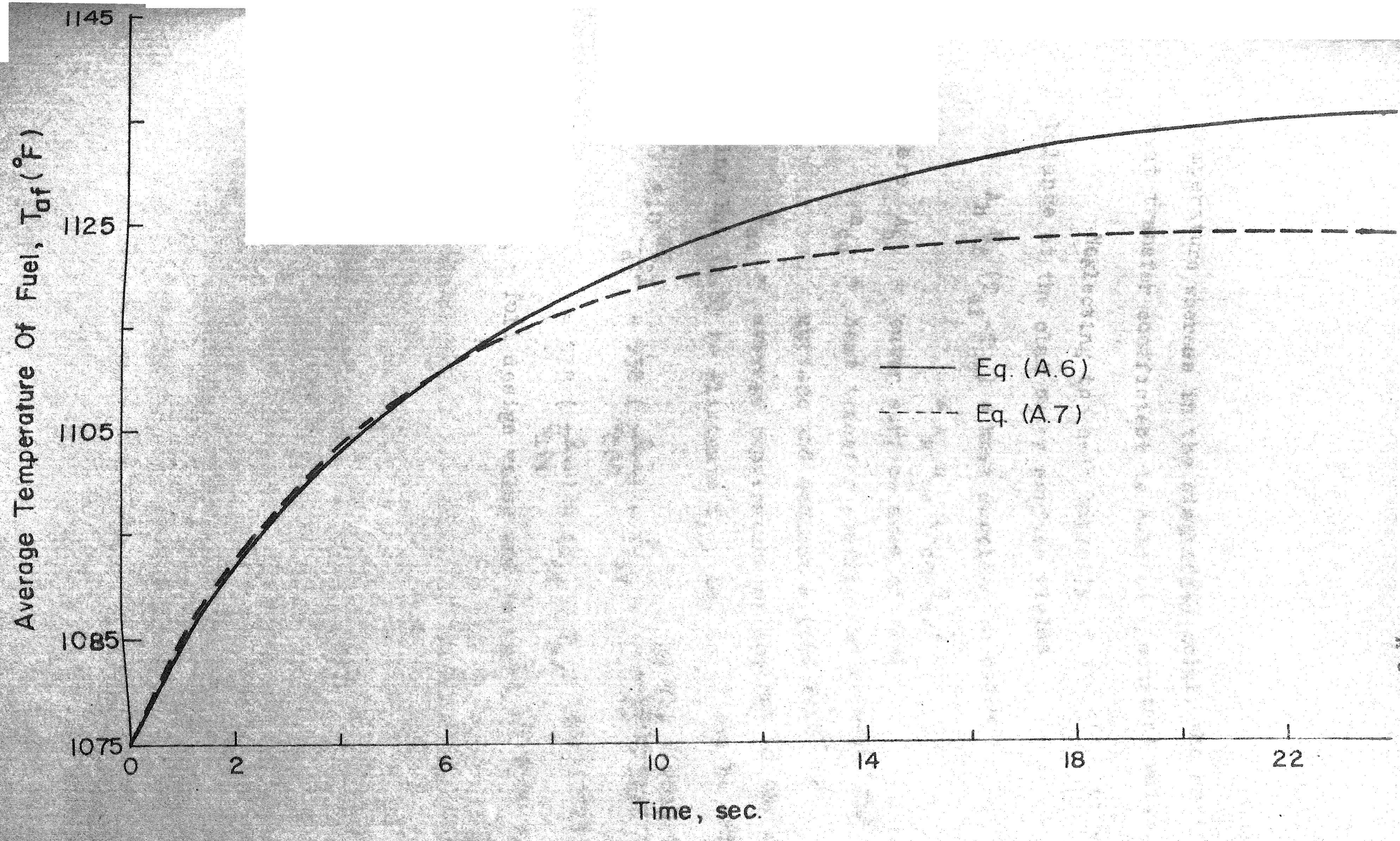

Fig. A.1 Response Of The Average Temperature Of The Fuel For a 10 % Step In Neutron Power

temperature whereas in the simplified model, the artificial heat transfer coefficient is held at a constant value.

Neglecting the heat capacity of the clad, an energy balance at the clad outer surface yields

$$A_h H_f (T_{af}-T_{co}) = \text{heat supplied to coolant}$$
$$= A_h h_{cc} (T_{co}-T_{ac}) \qquad (A.8)$$

where A_h = outer surface area of clad

h_{cc} = heat transfer coefficient between clad outer surface and coolant = 2640 W/ft^2 ·F

T_{ac} = average temperature of coolant in the reactor.

Using Eq. (A.8) to eliminate T_{co}, Eq. (A.7) can be written as

$$5.0 \frac{dT_{af}}{dt} = 538 \left(\frac{Q}{Q_{100}} \right) - \left(T_{af} - \frac{83\ T_{af} + 2640\ T_{ac}}{83 + 2640} \right)$$
$$= 538 \left(\frac{Q}{Q_{100}} \right) - (T_{af} - T_{ac}) (2640/2723) \qquad (A.9)$$

where Q_{100} - full design value and is equal to 7000 W/ft.

APPENDIX B

HEAT EXCHANGER DYNAMICS

For purposes of analysis, the heat exchanger can be divided into three portions: the recirculating leg performing the role of riser of the natural circulation loop and the boiling and preheat legs of the feedwater flow. For the recirculating leg, the following assumptions are made:

1. Temperature is dependent only on axial distance.
2. Density variation of the primary coolant (heavy water) with temperature is neglected.
3. All the primary coolant pumps are in operation at the rated speed. The velocity primary coolant in the heat exchanger is assumed to be constant i.e., it does not vary with time and with radial distance.
4. For the secondary coolant (water), nucleate boiling is assumed and the heat transfer from riser walls to water is taken to be proportional to the fourth power of the temperature difference between the wall and saturation temperature.

   $$\text{Heat flux (in Btu/hr ft}^2\text{)} = \text{constant} \times \exp(p/225)\,(T_{wall}-T_{sat})^4$$

where

p = pressure (psia)

T_{sat} = saturation temperature at pressure p (°F)

c - specific heat

H_1 - overall heat transfer coefficient between the heavy water and tube material based tube outer surface area

H_2 - boiling heat transfer coefficient $(10^6/60^4)\ \exp\ (p/225)\ (T_w - T_{sat})^3$

h - enthalpy of secondary coolant

w_r - mass flow rate of secondary coolant in the riser

u - velocity of primary coolant

s - outer perimeter of tube

z - axial distance (along the riser)

Using Eq. (B.1), Eq. (B.4) can be written as

$$A_2 \rho \frac{\partial h}{\partial t} + w_r \frac{\partial}{\partial z} h = s\ H_2\ (T_w - T_{sat}) \tag{B.5}$$

Equations (B.2), (B.3) and (B.5) predict the temperature and enthalpy distributions in the riser.

This set of partial differential equations is converted into a number of ordinary differential equations by eliminating the partial derivatives by finite (backward difference) difference technique. The heat exchanger is therefore divided into n sections, not necessarily of equal lengths, and the energy balance equations are written for the i^{th} section of length ℓ_i, by approximating

$$\frac{\partial T}{\partial z} \text{ by } \frac{T_i - T_{i-1}}{\ell_i} .$$

Then, the i^{th} section can be represented by the following equations:

$$(\rho c A_1)_{hw} \frac{dT_i}{dt} + (\rho c A_1)_{hw} u \frac{T_i - T_{i-1}}{\ell_i} = H_1 s (T_{wi} - T_i)$$

$$(\rho c A_3)_w \frac{dT_{wi}}{dt} = - H_1 s (T_{wi} - T_i) - H_2 s (T_{wi} - T_{sat})$$

$$A_2 \rho_i \frac{dh_i}{dt} + w_{ri} \frac{h_i - h_{i-1}}{\ell_i} = H_2 s (T_{wi} - T_{sat}) \qquad \text{(B.5a)}$$

The subscript i refers to the properties at the i^{th} section.

The mass flow rate w_{ri} has to be calculated by solving the equation for momentum balance for the entire natural circulation loop. But in this analysis, w_{ri} has been assumed to be a constant.

The above set of 3n equations are solved for steady state distributions at the full load operating conditions (design values), by making the time derivatives equal to zero. The resultant algebraic equations are solved with the following values:

Inlet temperature of heavy water = T_o = 560 ^oF

pressure p = 583 psia

w_r = 41.5 lb/sec

The value of n is increased until no significant improvement is observed in the calculated distribution. A value of 5 for n and ℓ_i = 2,2,4,6,6 ft. for i = 1,2,...5 are used.

From these steady state values, the single energy storage element approximation for the riser leg represented by outlet properties, is made. The heat transfer coefficient H_1 and the constant in H_2 are calculated such that both the n-segment approximation and the single energy-storage element approximation yield the same outlet properties for the primary and secondary coolants. These values of H_1 and H_2 are used to represent the riser leg and its dynamic behaviour is hence represented by the following equations:

$$\dot{T}_2 = 0.43\,(T_1 - T_2) - 1.61 \times 1.30\,(T_2 - T_{w2})$$

$$\dot{T}_{w2} = 1.74 \times \frac{1.61}{0.85}\,(T_2 - T_{w2}) - 8 \times 10^{-5} \times \frac{1.61}{0.85}\,\exp(p/225)\,(T_{w2} - T_{sat})^4$$

$$\dot{h}_r = \left[\frac{1.61}{0.85} \times 11 \times 10^{-3}\,\exp(p/225)\,(T_{w2} - T_{sat})^4 + w_r(h_f - h_r)\right] / (13.1\,\rho_r) \qquad \text{(B.6)}$$

where T_2, T_{w2}, h_r refer to the temperatures of heavy water and of wall of the tube and the enthalpy of the secondary coolant at the outlet of the riser and T_1 refers to the temperature of heavy water at inlet to the riser leg.

Transient responses of the single lumped system and the n-segment system are compared for various disturbances. The disturbances considered are:

1. a + 10 ^{o}F step in the temperature of heavy water at inlet to the riser, with the drum pressure at 583 psia, and
2. a +17 psia step in the drum pressure with the inlet temperature of heavy water at 560 ^{o}F. The responses

are shown in Figs. B.1 and B.2 and the agreement is quite good. Hence the dynamic behaviour of the riser leg will be represented by Eq. (B.6).

A similar study is made for the boiling leg. (The operating conditions are taken as: pressure = 583 psia, temperature of the heavy water at inlet = 510 ^oF, enthalpy of the feed water at inlet = h_f, and mass flow rate of the feed water = 8.7 lb/sec). The responses of single lumped system and 5-segment approximations are in good agreement for disturbances on pressure and inlet temperature of the heavy water and hence the boiling leg can be represented by:

$$\dot{T}_3 = 0.577\ (T_2-T_3) - 1.32x1.32\ (T_3-T_{w3})$$

$$\dot{T}_{w3} = \frac{1.32x1.74}{0.85}\ (T_3-T_{w3}) - 8x10^{-5} x\ \frac{1.32}{0.85}\ \exp\ (p/225)(T_{w3}-T_{sat})^4$$

$$\dot{h}_b = [\frac{1.32}{0.85}\ x\ 8.25x10^{-3}\ \exp\ (p/225)\ (T_{w3}-T_{sat})^4$$

$$- w_m\ (h_b-h_p)\ ]\ x\ 0.102/\rho_b \qquad (B.7)$$

where T_3, T_{w3}, h_b refer to the temperature of heavy water at outlet, the temperature of the wall at the outlet end of the boiling leg and enthalpy of feed water at its outlet end respectively. T_2 refers to temperature of heavy water entering the boiling leg and is equal to the temperature of heavy water leaving the riser leg. Similarly h_p refers to the enthalpy of the feed water entering the boiling leg and w_m refers to the mass flow rate of the feed water.

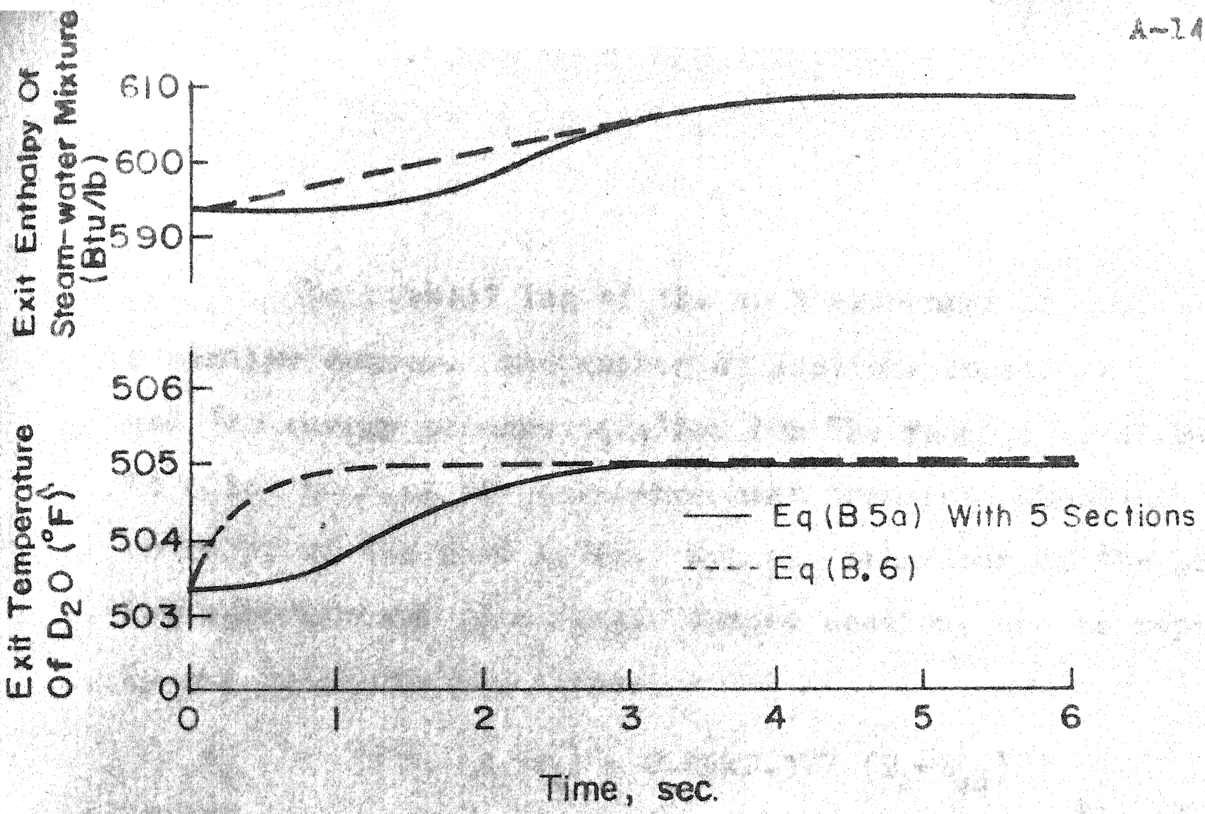

Fig. B.1 Response For a +10°F Step In The Inlet Temperature Of Heavy Water

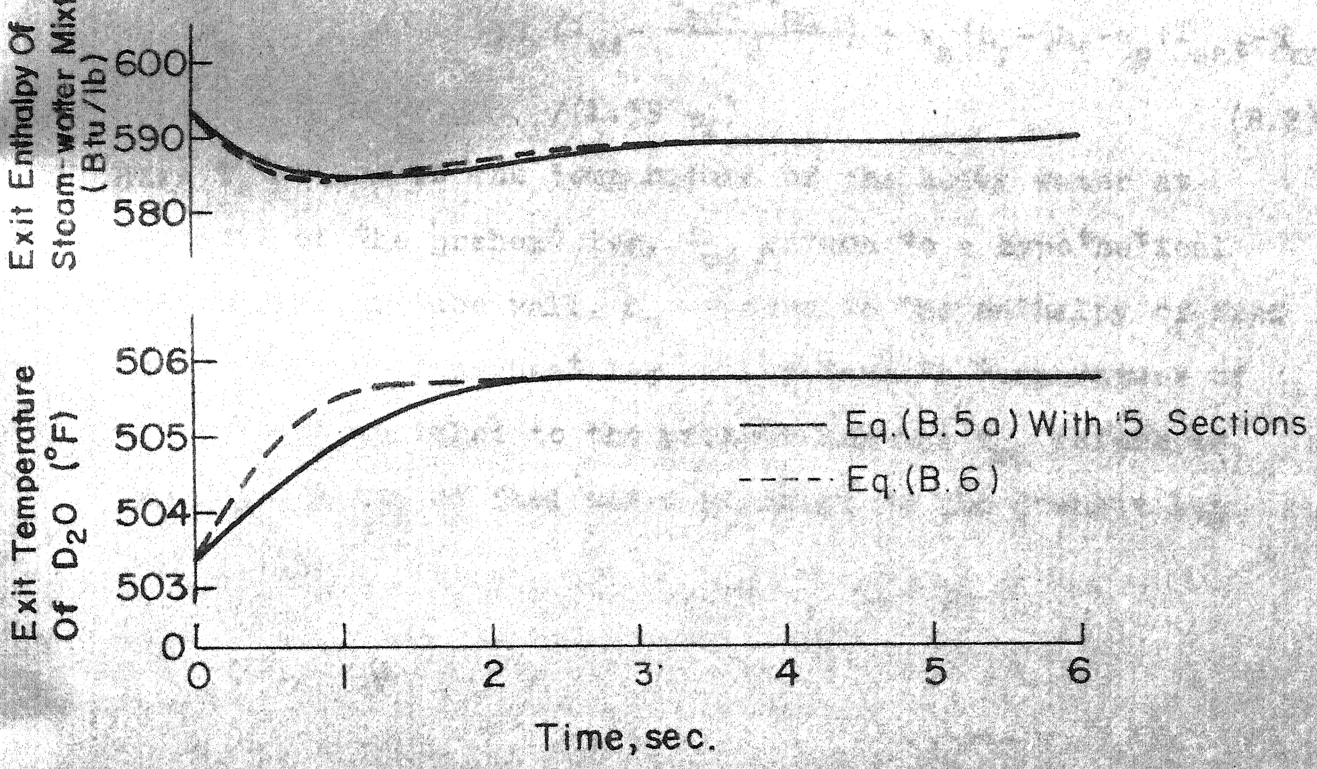

Fig. B 2 Response For a +17 psi Step In Steam Drum Pressure

The preheat leg of the heat exchanger is analysed in a similar manner. The number of sections considered, is ten and the energy balance equation for the feed water is modified to allow boiling or convective heat transfer depending on the enthalpy of the feed water. Dynamic behaviour of the preheat leg approximated as a single lumped section, can be represented by the following equations:

$$\dot{T}_4 = 1.76\ (T_3-T_4) - 0.85\text{x}0.377\ (T_4-T_{w4})$$

$$\dot{T}_{w4} = 1.74\text{x}0.377\ (T_4-T_{w4}) - 5.0\text{x}1.19\ \left(T_{w4} - \frac{T_{sat}+T_{mi}}{2}\right)$$

$$\dot{h}_p = \left[171\text{x}1.19\ \left(T_{w4}-\frac{T_{sat}+T_{mi}}{2}\right) - w_m(h_p-(h_f-c_p(T_{sat}-T_{mi}))\right] / (1.39\ \rho_f) \qquad (B.8)$$

Here T_4 refers to the temperature of the heavy water at the exit of the preheat leg, T_{w4} refers to a hypothetical temperature of tube wall, h_p refers to the enthalpy of feed water at exit of preheat leg. T_3 refers to temperature of heavy water at inlet to the preheat leg and T_{mi} refers to the temperature of feed water at inlet to the preheat leg.

APPENDIX C

STEAM DRUM

Steam drum is idealised as a cylindrical vessel with flow rates as shown in Fig. C.1

Symbols:

h = enthalpy

v = specific volume

M = mass in the drum

w = mass flow rate

p = drum pressure

T_{sat} = saturation temperature at pressure p

ρ = density

Subscripts:

f = saturated liquid

g = saturated vapour

mo = feed water from boiling leg

r = riser

d = downcomer

.(dot) indicates differentiation with respect to time t.

To evaluate the dynamic behaviour of the steam drum, the following assumptions are made:

1. 1. There is no spatial variation of temperature in the drum and the temperatures of liquid and vapour phase are assumed to be eq ual to saturation temperature. This assumption is valid since the feed water and the

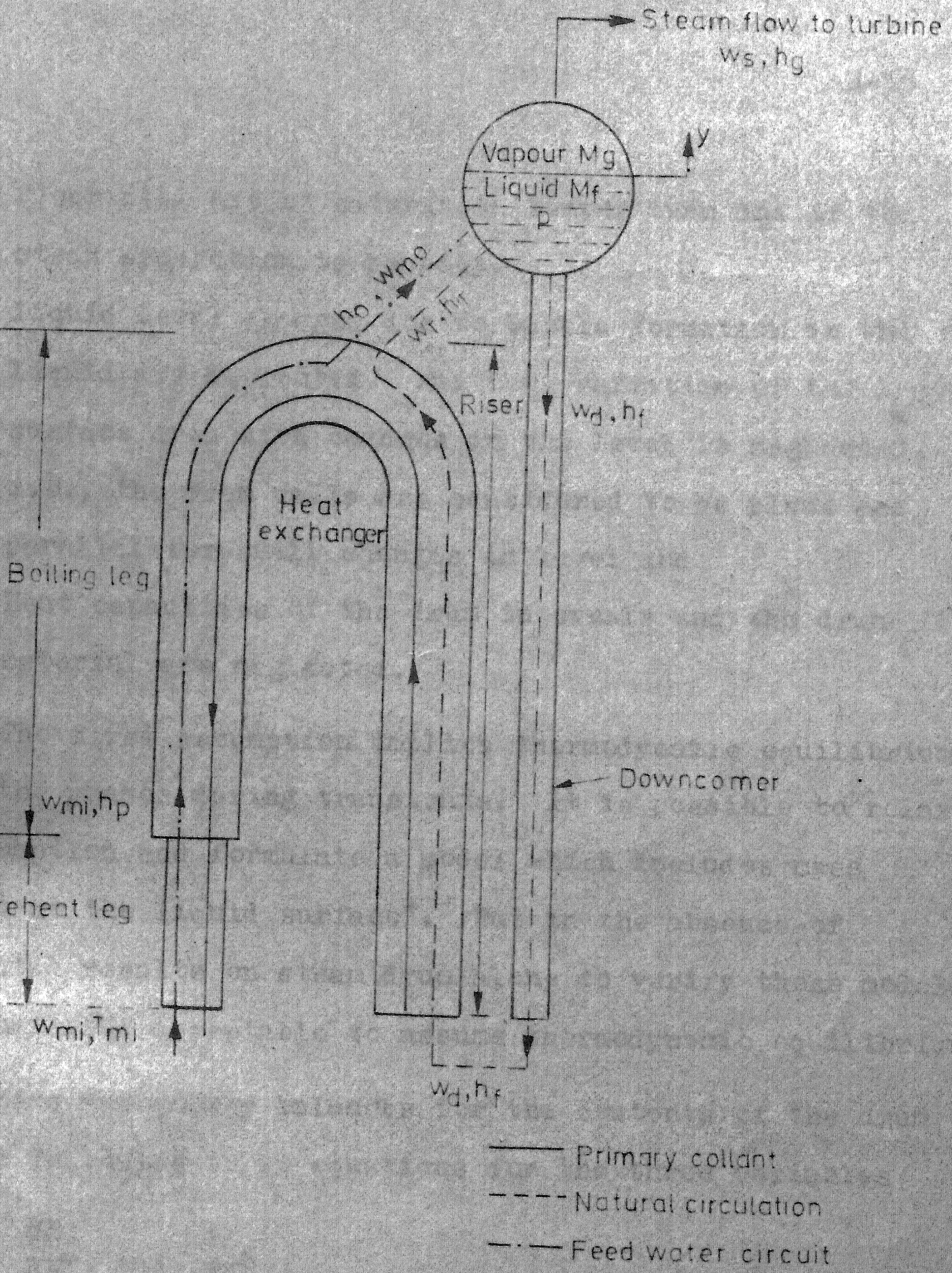

FIG. C.1 SCHEMATIC DIAGRAM OF THE MODEL FOR STEAM DRUM & NATURAL CIRCULATION LOOP

riser flow are at saturation temperature and if the steam separation is perfect.

2. Liquid level changes due to bubble formation in the liquid are neglected. Further, variation of the liquid surface area with changes in the level is neglected, i.e., the drum walls are considered to be plane and parallel for small changes in level and

3. Heat capacities of the drum internals and the drum material are neglected.

The first assumption implies thermodynamic equilibrium between the phases during transients. It is possible to relax this assumption and formulate a model which includes mass transport at the liquid surface[+]. But in the absence of experimental results on steam drum alone to verify those models, it is considered acceptable to assume thermodynamic equilibrium.

Mass and energy balances for the contents of the drum yield the following three equations for the three variables

$$\frac{dp}{dt}, \quad \frac{dM_f}{dt} \quad \text{and} \quad \frac{dM_g}{dt}$$

Mass balance:

$$\frac{dM}{dt} = \dot{M} = \dot{M}_f + \dot{M}_g = w_{mo} - w_s + w_r - w_d \qquad (C.1)$$

[+] 1. K.L. Chien et al., 'Dynamic Analysis of a Boiler', Trans. ASME, Vol.80, pp. 1809-1819, 1958

2. P.K. M'Pherson et al., 'Dynamic Analysis of a Nuclear Boiler', Proc. IME, Vol.180, pp. 417-427, 1965

3. P.K. M'Pherson, 'A Mathematical Model for Steam Drums', UKAEA Report AEEW-R 366, 1966

Energy balance:

$$\frac{d}{dt}(M_f h_f + M_g h_g) - \frac{d}{dt}(pV) = w_{mo} h_b + w_r h_r - w_d h_f - w_s h_g$$

Neglecting the second term on the left side, the above equation can be written as

$$h_f \dot{M}_f + \dot{M}_g h_g + \left(M_f \frac{dh_f}{dp} + M_g \frac{dh_g}{dp}\right) \frac{dp}{dt}$$

$$= w_{mo} h_b + w_r h_r - w_d h_f - w_s h_g \tag{C.2}$$

Volume of the drum is constant and this is expressed as

$$V = M_f v_f + M_g v_g = \text{constant}$$

Hence

$$\frac{dV}{dt} = \dot{M}_f v_f + \dot{M}_g v_g + \left(M_f \frac{dv_f}{dp} + M_g \frac{dv_g}{dp}\right) \frac{dp}{dt} = 0 \tag{C.3}$$

Equations (C.1), (C.2) and (C.3) are solved, after neglecting terms involving $\frac{dv_f}{dp}$ and $M_g \frac{dh_g}{dp}$ for the three unknowns viz., $\dot{p}$, $\dot{M}_f$, $\dot{M}_g$

$$\frac{dp}{dt} = B'/C' \tag{C.4}$$

and $$\frac{dM_f}{dt} = D'/E' \tag{C.5}$$

where $B' = w_s\, v_g\, h_{fg} - w_{mo}\, v_{fg}\, h_b - w_r\, v_{fg}\, (h_r - h_f)$

$+ w_{mo}\, (v_{fg}\, h_f - v_f\, h_{fg}) - (w_r - w_d)\, v_g\, h_{fg}$

$C' = M_g\, a_2\, (h_{fg} - v_{fg}\, F)$

$D' = w_{mo}(h_g - h_b) + w_r\, (h_g - h_r) - w_d\, h_{fg}$

$- [\, (w_{mo} - w_s) + (w_r - w_d)\,]\, v_g F'$

$$E' = h_{fg} - v_{fg} F'$$

$$F' = M_f a_1/M_g a_2$$

$$a_1 = \frac{dh_f}{dp}$$

and $$a_2 = \frac{dv_g}{dp}$$

Further,

$$M_f = \rho_f V_f$$

where V_f = volume of liquid in drum.

Measuring the liquid level y from the centre of the drum, V_f can be written for small variations of y about the centre of the drum, as

$$V_f = \left(\frac{1}{2} \cdot \frac{\pi d^2}{4} + y.d \right) \ell$$

where ℓ = longitudinal length of the drum

Hence $$M_f = \left(\frac{\pi d^2}{8} + yd\right) \ell \rho_f$$

and $$M_g = \left(\frac{\pi d^2}{8} - yd\right) \ell \rho_g$$

$$\frac{dM_f}{dt} = \ell d \rho_f \frac{dy}{dt}$$

This yields dy/dt as

$$\frac{dy}{dt} = \frac{1}{\rho_f \ell d} \cdot \frac{D'}{E'} \qquad (C.6)$$

and D' and E' are given in Eq. (C.5).

The above Eqs. (C.4) and (C.6) involve the flow rates from the riser and boiling legs and the flow rate to the downcomer. Following section develops expressions for these flow rates.

Natural circulation loop:

Steam drum, downcomer and riser (which is the shell of one leg of U-tube type heat exchanger) form the natural circulation loop. Mass flow rate in this can be studied by considering the momentum balance for this circuit+.

The following assumptions are made:

1. No boiling takes place in the downcomer and the fluid in the downcomer pipes is saturated liquid.
2. Vapour and liquid velocities in the riser are equal. Also thermodynamic equilibrium is assumed to exist between the phases.
3. Kinetic energy of the riser fluid entering the drum is dissipated in turbulence and the liquid in the drum is stationary (has no velocity towards the downcomer pipes).
4. Data for pressure drop due to internals of drum (like steam separator) are not available and hence that is not considered.

Momentum balance for downcomer pipe:

Available pressure head = pressure loss due to friction + gravitational head + entrance loss + inertia loss + head available due to kinetic energy at exit of downcomer.

+ K.L. Chien et al., 'Dynamic Analysis of a Boiler', Trans. ASME, Vol.80, pp. 1809-1819, 1958

(i.e)

$$(p-p_b) = f_d \left(\frac{\ell_d}{a_d^2\, d_d\, g\, \rho_f}\right) w_d^2 - \rho_f \ell_d + \frac{k_d}{a_d^2 \rho_f . g} w_d^2$$

$$+ \frac{\ell_d}{a_d\, g} \dot{w}_d + \frac{w_d^2}{g\, a_d^2 \rho_f} \qquad (C.7)$$

where f_d = frictional coefficient for downcomer

k_d = entrance loss coefficient for downcomer

ℓ_d = length of downcomer pipe

a_d = area (flow area) of downcomer

d_d = diameter of downcomer

p = drum pressure

p_b = pressure (static) at the bottom of downcomer pipe.

Momentum balance for riser leg can be written as

$$p_b - p = f'_r \left(\frac{f_r \ell_r}{a_r^2\, d_r\, g} \frac{w_r^2}{f}\right) + \int_0^{\ell_r} \rho dz + \frac{k_r w_r^2}{g\, a_r^2\, \rho_r} +$$

$$\frac{\ell_r}{a_r\, g} \dot{w}_r + \left(\frac{w_r^2}{\rho_r a_r^2\, g} - \frac{w_d^2}{\rho_f\, a_d^2\, g}\right) \qquad (C.8)$$

where k_r = exit loss coefficient for riser

f_r' = frictional loss coefficient for two phase flow in the riser

= $1+2400\,(x/p)^{0.96}$ p in kg/cm^2+, x - quality of flow

f_r = frictional loss coefficient for single phase flow

Figure C.2 shows the variation of the density of riser fluid with axial distance. In this analysis $\int_0^{\ell_r} \rho dz$ has been

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+ K.M. Becker et al., Aktiebolaget Atomenergi Report AE-69, 70 and 85, 1962

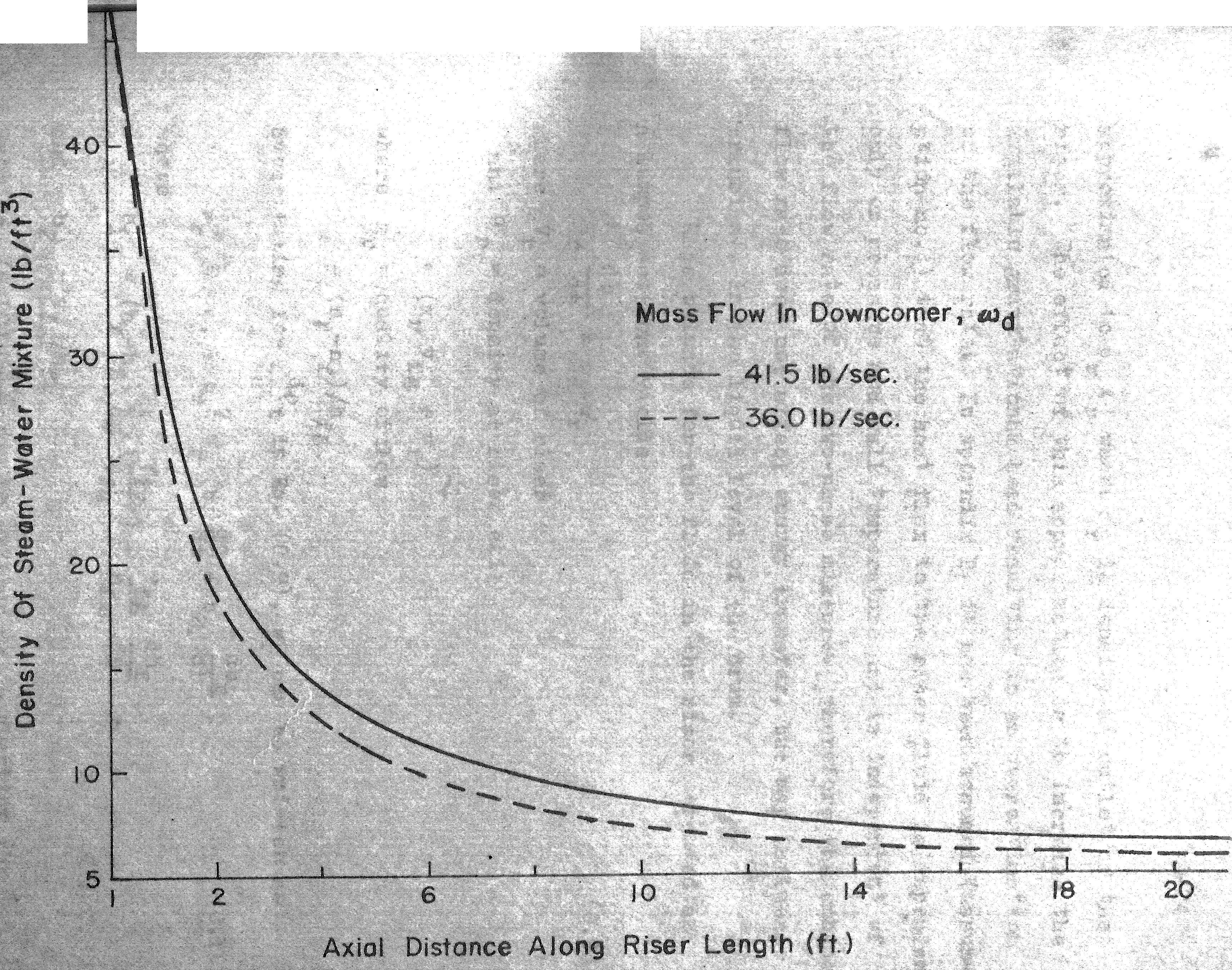

Fig. C.2 Variation Of Riser Fluid Density With Axial Distance

approximated to $\rho_r\,\ell_r$ where ρ_r is density at outlet of the riser. The effect of this approximation is to increase the available gravitational head resulting in an overestimation of the flow rate. In Apprndix B, it has been assumed (assumption no.4) that the heat flux to the riser fluid is dependent only on pressure and wall temperature and is independent of the flow rate of the two-phase mixture. Therefore the enhanced flow rate does not affect energy transfer, but may influence transients in the liquid level of the drum.

Mass balance for the fluid in the riser treated as a lumped section. yields

$$V_r \frac{d\rho_r}{dt} = w_d - w_r \tag{C.9}$$

where V_r = volume of riser

and ρ_r = density at riser exit

$$= (x_r v_{fg} + v_f)^{-1}$$

where x_r = quality of flow

$$= (h_r - h_f)/h_{fg}$$

Substituting for $\frac{d\rho_r}{dt}$ in Eq. (C.9), w_r can be written as

$$w_r = w_d + \rho_r^2 V_r \left(b_1 \frac{dp}{dt} + b_2 \frac{dh_r}{dt}\right) \tag{C.10}$$

where

$$b_1 = (h_r - h_f) \frac{d}{dp} \left(\frac{v_{fg}}{h_{fg}}\right) - \frac{v_{fg}}{h_{fg}} \frac{dh_f}{dp}$$

and $b_2 = v_{fg}/h_{fg}$

From Eq. (C.10), $\frac{dw_r}{dt}$ can be written as, neglecting the second derivatives,

$$\frac{dw_r}{dt} = \dot{w}_d - z \qquad (C.11)$$

where

$$z = 2V_r \rho_r^3 (b_1 \dot{p} + b_2 \dot{h}_r)^2 - 2V_r \rho_r^2 a_1 (\dot{h}_r - a_2 \dot{p}) \dot{p}$$

$$a_1 = \frac{d}{dp} (v_{fg}/h_{fg}) = - 0.1681x10^{-5}$$

$$a_2 = \frac{d}{dp}(h_f) = 0.2178$$

Combining Eqs. (C.7), (C.8), (C.10) and (C.11), to eliminate p_b and p and substituting values for physical dimensions and constants, the flow rate w_d can be written as

$$\dot{w}_d = [\,20\ (\rho_f - \rho_r) - \frac{2.21}{\rho_f} w_d^2 - \frac{w_r^2}{\rho_r} \{1.036 + 0.274\ x\ (1+2400(\frac{x_r}{p})^{.96}) \frac{\rho_r}{\rho_f}\} + 0.95\ z]\ /5.51$$

$$(C.12)$$

where z is defined in Eq. (C.11) and $\dot{w}_r$ is given by Eq. (C.10).

The flow rate entering the steam drum from the boiling leg is obtained by mass balance for the boiling leg and is, similar to Eq. (C.10) written as

$$w_{mo} = w_{mi} + V_b \rho_b^2 (b_3 \dot{p} + b_4 \dot{h}_b) \qquad (C.13)$$

where

$$b_3 = (h_b - h_p) \frac{d}{dp} (v_{fg}/h_{fg}) - \frac{v_{fg}}{h_{fg}} \frac{dh_f}{dp}$$

$$b_4 = v_{fg}/h_{fg}$$

and w_{mi} = flow rate entering the boiling leg which is assumed by neglecting the density changes in the preheat leg, to be

equal to the mass flow entering the preheat leg.

Equations (C.4), (C.11) and (C.12) are combined to yield, after rearranging the terms

$$\frac{dp}{dt} = (A'-B'-C'-D'b_2\,\dot{h}_r)/(E'+F'b_3+G'b_1) \tag{C.14}$$

where
$$A' = w_s\, v_g\, h_{fg}$$
$$B' = (v_{fg}\, h_b - v_{fg}\, h_f + v_f\, h_{fg})(w_{mi} + V_b\, \rho_b^2\, b_4\, \dot{h}_b)$$
$$C' = (h_r-h_f)\, v_{fg}(w_d+V_r\rho_r^2\, b_2\, \dot{h}_r)$$
$$D' = v_f\, h_{fg}\, V_r\rho_r^2$$
$$E' = M_g\, (dv_g/dp)\, (h_{fg}-v_{fg}\, E'')$$
$$E'' = \frac{dh_f}{dp} M_f \,/\, (M_g \frac{dv_g}{dp})$$
$$F' = (v_{fg}\, h_b - v_{fg}\, h_f + v_f\, h_{fg})\, V_b\, \rho_b^2$$
$$G' = ((h_r-h_f)\, v_{fg} + v_f\, h_{fg})\, V_r\, \rho_r^2$$

and b_1, b_2, b_3 and b_4 are defined in Eqs. (C.10) and (C.13)

Mass and energy balance for the riser and boiling legs, represented as a single lumped energy storage systems, yield (Appendix B)

$$\dot{h}_r = [\frac{1.61}{0.85} \times 0.011 \exp(p/225)\, (T_{w2}-T_{sat})^4 + w_d(h_f-h_r)] /(13.1\, \rho_r) \tag{C.15}$$

and

$$\dot{h}_b = [\frac{1.32}{0.85} \times 0.00825 \exp(p/225)\, (T_{w3}-T_{sat})^4 - w_{mi}(h_b-h_p)] \times 0.102/\rho_b \tag{C.16}$$

Similarly, the drum liquid level can be represented by

$$\frac{dy}{dt} = (A'+B'-C'-D')/(E'xF') \tag{C.17}$$

where

$$A' = w_{mo}\ (h_g - h_b)$$

$$w_{mo} = w_{mi} + V_b\ \rho_b^2\ (b_4\ \dot{h}_b + b_3\ \dot{p})$$

$$B' = w_r\ (h_g - h_r)$$

$$w_r = w_d + V_r\ \rho_r^2\ (b_2\ \dot{h}_r + b_1\ \dot{p})$$

$$C' = w_d\ h_{fg}$$

$$D' = ((w_{mo} - w_s) + (w_r - w_d))\ v_g . E''$$

$$E' = h_{fg} - v_{fg}\ E''$$

$$E'' = \frac{dh_f}{dp}\ M_f / (M_g\ \frac{dv_g}{dp})$$

$$F' = \rho_f\ \ell d$$

and b_1, b_2, b_3 and b_4 are defined in Eqs. (C.10) and (C.13).

Equations (C.12), (C.15), (C.16), (C.14) and (C.17) describe the dynamic behaviour of the steam drum and natural circulation loop.

Further, thermodynamic properties needed for the analysis are assumed as a linear function of pressure over the range of 450-700 psia and are given by

$$h_f = 340.24 + 0.2178\ p$$

$$h_g = 1211 - 0.01336\ p$$

$$T_{sat} = 372.847 + 0.1878\ p$$

$$v_g = 1.6782 - 0.0014933\ p$$

$$v_f = 0.01783 + 0.3829 \times 10^{-5} p$$

Figure C.3 shows a comparison of the values calculated using the above equations with those provided by Keenan and Keyes.+

+ J.H. Keenan and F.G. Keyes, 'Thermodynamic Properties of Steam', John Wiley and Sons, Inc., 1936.

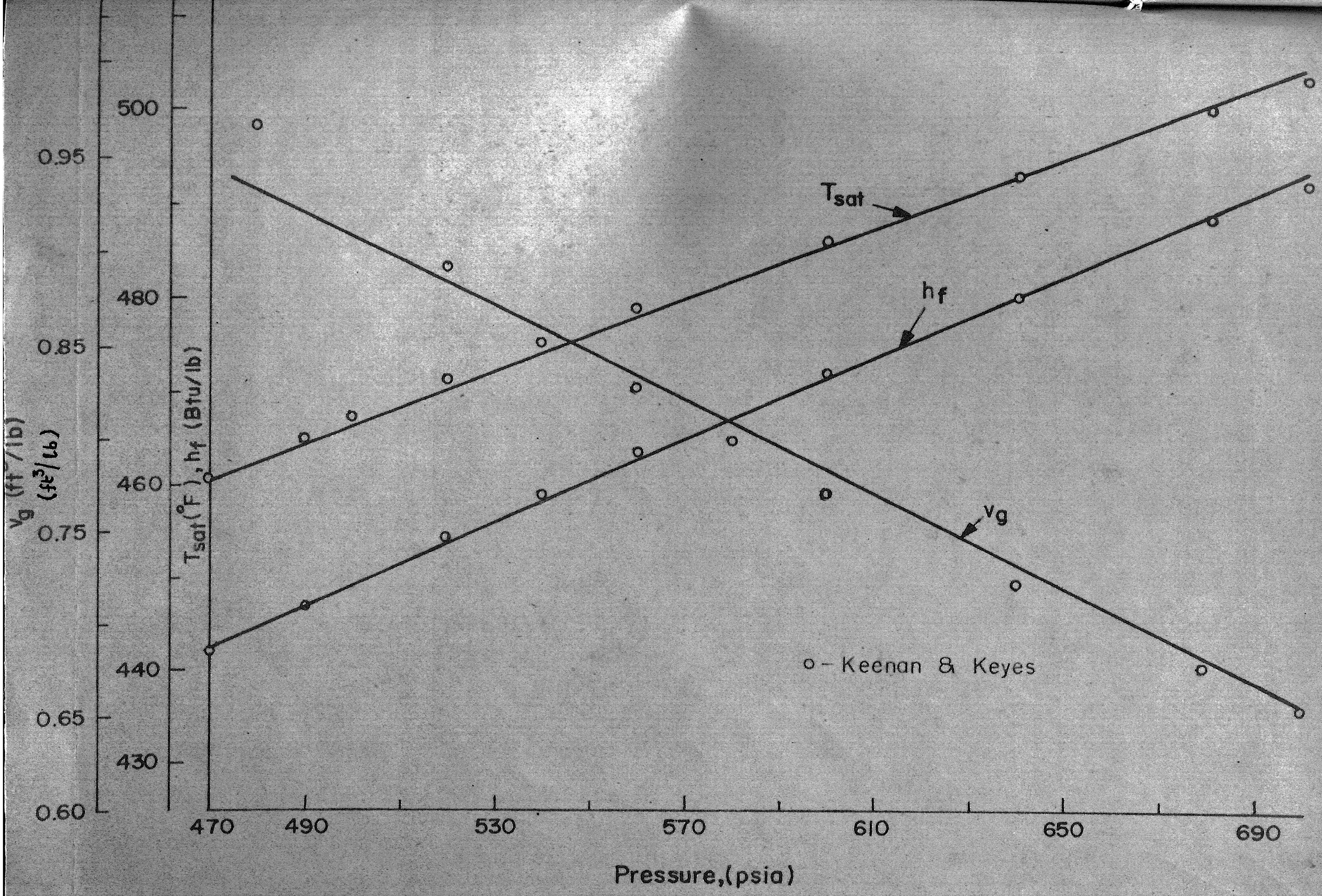

Fig. C. 3 Variation Of Thermodynamic Properties Of Water With Pressure.

APPENDIX D

CONTROLLABILITY

A system is said to be completely state controllable if it is possible to transfer any given initial state $\underline{x}(t_o)$ to any desired final state $\underline{x}(t_f)$ in a finite time interval $t_o < t < t_f$ with an unconstrained control vector $\underline{u}(t)$.

For a linear time invariant system given by

$\dot{\underline{x}} = \underline{A}\ \underline{x} + \underline{Bu}$

$\underline{x}$ is a (nxl) vector of state variables

$\underline{u}$ is a (mxl) vector of control variables

$\underline{A}$, $\underline{B}$ are matrices of appropriate size

the complete state controllability implies+ that the composite W-matrix

$$\underline{W} = (\underline{B} \vdots \underline{AB} \vdots \underline{A}^2\underline{B} \vdots \cdots \vdots \underline{A}^{n-1}\underline{B}) \text{ is of rank n.}$$

+ K. Ogata, 'State Space Analysis of Control System' Prentice Hall, 1967

APPENDIX E

LINEAR REGULATOR THEORY

In the linear regulator problem+, we have a linear differential system

$$\dot{\underline{x}} = \underline{A}(t)\underline{x} + \underline{B}(t)\underline{u} \; , \; \underline{x}(t_o) = \underline{x}_o \tag{E.1}$$

where $\underline{x}$ is a vector of n variables, $\underline{u}$ is a vector of m variables, $\underline{A}$ and $\underline{B}$ are matrices of appropriate sizes. and wish to find the control which minimizes the cost function (for t_f fixed)

$$J = \frac{1}{2}\, \underline{x}^T(t_f)\; \underline{S}\; \underline{x}(t_f) + \frac{1}{2} \int_{t_o}^{t_f} (\underline{x}^T(t)\; \underline{Q}\; \underline{x}(t) + \underline{u}^T\,(t)\; \underline{R}\; \underline{u}(t))\, dt \tag{E.2}$$

The matrices $\underline{Q}$, $\underline{R}$ and $\underline{S}$ are assumed to be symmetric. We may obtain the solution to this problem via the Pontryagin's maximum principle or the Hamilton-Jacobi equation. Using the former method, the Hamiltonian is written as

$$H\,[\underline{x}(t),\, \underline{u}(t),\, \lambda(t),\, t] = \frac{1}{2}\, \underline{x}^T\, \underline{Q}\, \underline{x} + \frac{1}{2}\, \underline{u}^T\, \underline{R}\, \underline{u} + \underline{\lambda}^T\, \underline{A}\, \underline{x} + \underline{\lambda}^T\, \underline{B}\, \underline{u} \tag{E.3}$$

Application of the maximum principle requires that, for an optimum control,

$$\frac{\partial H}{\partial \underline{u}} = \underline{0} = \underline{R}(t)\; \underline{u}(t) + \underline{B}^T(t)\; \underline{\lambda}(t) \tag{E.4}$$

and
$$\frac{\partial H}{\partial \underline{x}} = -\dot{\underline{\lambda}} = \underline{Q}(t)\; \underline{x}(t) + \underline{A}^T(t)\; \underline{\lambda}(t) \tag{E.5}$$

with the terminal condition

$$\underline{\lambda}(t_f) = \underline{S}\; \underline{x}(t_f) \tag{E.6}$$

+ A.P. Sage, 'Optimum Systems Control', Prentice-Hall, 1968

Thus $\underline{u}(t) = -\underline{R}^{-1}(t)\ \underline{B}^T(t)\ \underline{\lambda}(t)$ (E.7)

To convert this to a closed loop control, it is assumed that the solution for $\underline{\lambda}$ is similar to Eq.(E.6).

$$\underline{\lambda}(t) = \underline{P}(t)\ \underline{x}(t) \qquad \text{(E.8)}$$

Substituting this relation into Eq. (E.1) and (E.7), Eq.(E.1) can be written as

$$\dot{\underline{x}} = \underline{A}(t)\ \underline{x}(t) - \underline{B}(t)\ \underline{R}^{-1}(t)\ \underline{B}^T(t)\ \underline{P}(t)\ \underline{x}(t) \qquad \text{(E.9)}$$

Also from Eq. (E.8) and (E.5), $\dot{\underline{\lambda}}$ can be written as

$$\begin{aligned}\dot{\underline{\lambda}} &= \dot{\underline{P}}\ \underline{x}(t) + \underline{P}\ \dot{\underline{x}} \\ &= -\underline{Q}(t)\ \underline{x}(t) - \underline{A}^T(t)\ \underline{P}(t)\ \underline{x}(t) \qquad \text{(E.10)}\end{aligned}$$

By combining Eq.(E.9) and (E.10), we have

$$\begin{aligned}[\dot{\underline{P}} + \underline{P}(t)\ \underline{A}(t) + \underline{A}^T(t)\ \underline{P}(t) - \underline{P}(t)\ \underline{B}(t)\ \underline{R}^{-1}(t)\ \underline{B}^T(t)\ \underline{P}(t) \\ + \underline{Q}(t)]\ \underline{x}(t) = \underline{0} \qquad \text{(E.11)}\end{aligned}$$

Since this must hold for all nonzero $\underline{x}(t)$, the term multiplying $\underline{x}(t)$ must be zero. Thus the P matrix, which is an n x n symmetric matrix and which has n(n+1)/2 different terms, must satisfy the matrix Riccati equation

$$\begin{aligned}\dot{\underline{P}} = -\underline{P}(t)\ \underline{A}(t) - \underline{A}^T(t)\ \underline{P}(t) + \\ \underline{P}(t)\ \underline{B}(t)\ \underline{R}^{-1}(t)\ \underline{B}^T(t)\ \underline{P}(t) - \underline{Q}(t) \qquad \text{(E.12)}\end{aligned}$$

with a terminal condition given by

$$\underline{P}(t_f) = \underline{S} \qquad \text{(E.13)}$$

Thus we may solve the matrix Riccati equation backward in time from t_f to t_o and store the matrix

$$\underline{K}(t) = -\underline{R}^{-1}(t)\ \underline{B}^T(t)\ \underline{P}(t) \qquad \text{(E.14)}$$

A closed loop control is then obtained from

$$\underline{u}(t) = \underline{\underline{K}}(t)\ \underline{x}(t) \qquad (E.15)$$

which yields the desired state feedback control law.

APPENDIX F

FUNCTION MINIMISATION

Fletcher-Reeves method+ is one of the methods used for minimising a function, where a large amount of local information about the function in terms of the gradient of the function being minimised, is utilised.

The problem of function minimisation can be stated as:

To find the vector $\underline{x}$ such that the function $F(\underline{x})$ is a minimum where there are no restrictions on the choice of $\underline{x}$.

The gradient of the function F lies in the direction of greatest rate of change of function and has that rate of change as its magnitude. The gradient, which is a vector and denoted by ∇F, is defined as

$$\nabla F \equiv \underline{G} \equiv \left(\frac{\partial F}{\partial x_1}, \frac{\partial F}{\partial x_2} \ldots \frac{\partial F}{\partial x_n}\right) \tag{F.1}$$

With this definition, Fletcher-Reeves method can be stated as: Selecting an initial choice of $\underline{x}$ as $\underline{x}_o$, the gradient $\underline{G}_o$ at $\underline{x}_o$ is calculated. The direction in which the function will decrease, is then

$$\underline{S}_o = - \underline{G}_o$$

The function F is minimised in this direction such that at

$$\underline{x}_1 = \underline{x}_o + \alpha' \underline{S}_o \tag{F.2}$$

$F(\underline{x}_1) < F(\underline{x}_o + \alpha \underline{S}_o)$ for any α. At the new point $\underline{x}_1$, the gradient $\underline{G}_1$ is calculated and the new direction for function minimisation is written as

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+ R.L. Fox, 'Optimisation Methods for Engineering Design', Addison-Wesley Pub. Co., 1971

$$\underline{S}_1 = - \underline{G}_1 + \frac{|\underline{G}_1|^2}{|\underline{G}_0|^2} \underline{S}_0 \qquad (F.3)$$

and the function minimisation in the direction $\underline{S}_1$ is then carried out to find α' and the new $\underline{x}$. Thus at the end of i^{th} iteration

$$\underline{S}_{i+1} = - \underline{G}_{i+1} + \frac{|\underline{G}_{i+1}|^2}{|\underline{G}_i|^2} \underline{S}_i \qquad (F.4)$$

and the procedure is repeated till the required accuracy is obtained.

α' in Eq. (F.2) is obtained by using the method of quadratic interpolation. Considering any vector $\underline{S}$ and the initial point $\underline{x}_o$ with function value F_1, a new vector $\underline{x}$ is defined as

$$\underline{x} = \underline{x}_o + \alpha \ \underline{S}.$$

The parameter α is then adjusted such that

$$F_1 \equiv F(\underline{x}_o)$$

$$F_2 \equiv F(\underline{x}_o + \alpha \ \underline{S}) < F_1$$

and $$F_3 \equiv F(\underline{x}_o + 2\alpha \ \underline{S}) > F_1$$

Approximating the function $F(\alpha)$ by $H(\alpha)$ which is a quadratic in α, i.e., $H(\alpha) = a + b\alpha + c\alpha^2$, leads to the minimum of $H(\alpha)$ at α'

where $$\alpha' = \frac{4F_2 - 3F_2 - F_3}{4F_2 - 2F_1 - 2F_3} \alpha$$

$\underline{x}'$ calculated as $\underline{x}_o + \alpha' \ \underline{S}$, is then taken as the minimum of $F(\underline{x})$ in the direction $\underline{S}$.